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Formulation ofSolutions of a Special Class of Standard Bi-quadratic Congruence of Composite Modulus-an Integer- Multiple of Power of an Odd Prime.

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ABSTRACT
In this paper, solutions of a special class of standard bi-quadratic congruence of composite modulus-an integer multiple of power of an odd prime- is formulated. The formula established is tested and verified true by citing numerical examples. The merit of the paper is the establishment of the formula. One need not use the preferred Chinese Remainder Theorem- a time consuming method.
Key–words: Bi-quadratic congruence, Composite modulus, Chinese Remainder Theorem, Prime-power integer, Formulation.

1. INTRODUCTION

A congruence of the type $x^4 \equiv a \pmod{m}$, is called a standard bi-quadratic congruence of prime or composite modulus if m is prime or composite integer.
The values of x are the solutions of the congruence.
The author has formulated many standard bi-quadratic congruence of prime and composite modulus[1], [2], [3], [4], [5], [6], [7], [8], [9]. Here is another bi-quadratic congruence of composite modulus (special type) is considered for the formulation.
It is of the type:
$x^4 \equiv p^4 \ (mod\ b.p^n), n \geq 4,$ and b an integer, p an odd prime.

2. REVIEW OF LITERATURE

It is found that no attempt had been made for formulation of the congruence by earlier mathematicians. No special literature is found. Readers preferred to use Chinese Remainder Theorem to solve the congruence, which takes a long time. Thomas Koshy had mentionedthe definition of standard bi-quadratic congruence but no method of solutions is discussed nor mention any formulation of the congruence [10].

3. NEED OF THIS RESEARCH

As no direct formulation is found in the literature of mathematics, readers must need a new time-saving method or formulation of solutions of the congruence.For the time saving purpose, the author has tried his best to formulate the congruence and his efforts are presented in this paper. This is the need of the research.

4. PROBLEM STATEMENT

"To formulate the congruence:
$x^4 \equiv p^4$ (mod $b.p^n$), $n \geq 4$,and b an integer".

ANALYSIS & RESULT
The congruence under consideration is:
$x^4 \equiv p^4\ (mod\ b.p^n), m \geq 4.$
For the solutions, consider, $x = b.p^{n-3}k \pm p; k = 0, 1, 2, 3, \dots \dots \dots$
Then,$x^4 = (p^{n-3}.k\ \pm p)^4$

$$= b^4.p^{4n-12}.k^4 \pm 4.b^3.p^{3n-9}.k^3.p + \frac{4.3}{1.2}.b^2.p^{2n-6}k^2.p^2 \pm \frac{4.3.2}{1.2.3}.b.p^{n-3}k\ \ .p^3 + p^4$$

$= p^4$ +b.$p^n(\dots\dots)$

$$\equiv p^4\ (mod\ b.p^n)$$

Thus, $x \equiv b.p^{n-3}k \pm p\ (mod\ b.p^n)$ are the solutions for$k = 0, 1, 2, 3, \dots \dots$
But if $k = p^3$, then $x \equiv b.p^{n-3}.p^3 \pm p(mod\ b.p^n)$

$$\equiv b.p^n \pm p(mod\ b.p^n)$$

$\equiv o \pm p\ (mod\ b.p^n)$, which is the same solution as for $k = o$.
For $k = p^3 + 1$, it can be easily seen that the solution is the same as for $k = 1$ as:

$$x \equiv b.p^{n-3}(p^3 + 1) \pm p\ (mod\ b.p^n)$$
$$\equiv \{b.p^{n-3}p^3 +\ b.p^{n-3}\} \pm p\ (mod\ b.p^n)$$
$$\equiv \{b.p^n +\ b.p^{n-3}\} \pm p\ (mod\ b.p^n)$$
$$\equiv (0 + b.p^{n-3}\ )\} \pm p\ (mod\ b.p^n)$$

$\equiv b.p^{n-3}.1 \pm p\ (mod\ b.p^n)$.

Thus, all the solutions can be given by $x \equiv b.p^{n-3}k \pm p\ (mod\ b.p^n)$;

$$k = 0,1,2, \dots.(p^3 - 1).$$

Therefore, the above congruence has $2p^3$ solutions and the solutions are

$$x \equiv (p^{n-3}k \pm p)(mod\ b.p^n); k = 0, 1, 2, \dots \dots..(p^3 - 1).$$

If one take $b = 1$, the congruence under consideration reduces to
$x^4 \equiv p^4\ (mod\ p^n)$. In this case, we definitely have $n \geq 5$.
And the solutions are given by

$$x \equiv (p^{n-3}k \pm p)(mod\ p^n); k = 0, 1, 2, \dots \dots..(p^3 - 1).$$

These are $2p^3$ solutions of the congruence.

ILLUSTRATIONS
Consider the congruence $x^4 \equiv 81\ (mod\ 1215)$
Here, $1215 = 5.243 = 5.3^5\ \ with\ n = 5, b = 5, and\ p = 3.$
The congruence can also be written as $x^4 \equiv 3^4(mod\ 5.3^5)\ \ with\ p = 3.$
Thus the congruence has $2p^3 = 2.27 = 54$ solutions, given by

$$x \equiv (bp^{n-3}k \pm p)(mod\ b.p^n), k = 0, 1, 2, \dots \dots \dots (p^3 - 1).$$
$$\equiv 5.3^{5-3}k \pm 3\ \ (mod\ 5.3^5); k = 0, 1, 2, \dots \dots.(27 - 1) = 26.$$
$$\equiv 5.3^2k \pm 3\ (mod\ 5.243)$$
$$\equiv 45k \pm 3\ (mod\ 1215); k = 0, 1, 2, \dots \dots, 26.$$
$$\equiv 0 \pm 3; 45 \pm 3; 90 \pm 3; 135 \pm 3; 180 \pm 3; \dots \dots \dots \dots \dots \dots \dots.;$$
$$1125 \pm 3; 1170 \pm 3\ (mod\ 1215)$$
$$\equiv 3, 1212; 42, 48; 87, 93; 132, 138; 177, 183; \dots. \dots. \dots.$$
$$1122, 1128; 1167, 1173\ (mod\ 1215).$$

These are the 54 incongruent solutions of the congruence.
Let us consider another example. $x^4 \equiv 625\ (mod\ 2500)$

The congruence can also be written as $x^4 \equiv 5^4 (mod\ 4.5^4)$.
Here, $2500 = 4.625 = 4.5^4\ with\ p = 5, b = 4, n = 4$.
Thus, the congruence has $2.p^3 = 250$Solutions given by

$$x \equiv b.p^{n-3}k.\pm p\ \ (modb.p^n); k = 0, 1,2, \ldots.., (p^3 - 1).$$

$\equiv\ 4.5^{4-3}k.\pm 5\ \ (mod\ 4.5^4); k = 0,1,2, \ldots\ldots.., (125 - 1).$

$$\equiv\ 20k \pm 5\ \ (mod\ 2500); k = 0, 1, 2, \ldots\ldots., 124.$$

$\equiv 0 \pm 5; 20 \pm 5; 40 \pm 5\ ; 60 \pm 5; 80 \pm 5; \ldots\ldots\ldots\ldots\ldots\ldots, 2480 \pm 5\ (mod\ 2500).$
≡5, 2495; 15, 25; 35, 45; 55, 65; 75, 85;…………;2475, 2485 (mod 2500).
These are required two hundred &fifty solutions.
Consider one more example: $x^4 \equiv 2401\ (mod\ 16807)$
It can be written as $x^4 \equiv 7^4\ (mod\ 7^5)$ with $p = 7, n = 5\&\ b = 1$.
It has $2\boldsymbol{p^3} = \mathbf{2.343} = \mathbf{686}$ incongruent solutions.
These are given by

$$\boldsymbol{x} \equiv \boldsymbol{p}^{n-3}k.\pm p\ \ (mod\ p^n); k = 0, 1,2, \ldots.., (p^3 - 1).$$

$\equiv\ 7^{5-3}k.\pm 7\ \ (mod\ 7^5); k = 0,1,2, \ldots\ldots.., (343 - 1).$

$$\equiv 49k \pm 7\ \ (mod\ 16807); k = 0, 1, 2, \ldots\ldots., 342.$$

$\equiv 0 \pm 7; 49 \pm 7; 98 \pm 7;\ 147 \pm 7; \ldots\ldots\ldots\ldots\ldots\ldots, 16758 \pm 7\ (mod\ 16807).$
≡7, 16800; 42, 56; 91, 105; 140, 154; …………; 16751, 16765 (mod 16807).
These are the 686 incongruent solutions of the said congruence.

CONCLUSION

Thus, it can be concluded that the solutions of the standard bi-quadratic congruence of composite modulus of the type: $x^4 \equiv p^4\ (mod\ b.p^n)$, with p an odd prime integer, is formulated. The solutions are given by

$x \equiv (bp^{n-3}k \pm p)(mod\ b.p^n), k = 0, 1, 2, \ldots\ldots\ldots (p^3 - 1)$, Which are $2p^3$ in number.

The established formula is tested and verified true by solving some examples. It is a very quick method to find all the solutions.

MERIT OF THE PAPER

A very simple formulation is made. The solutions can be calculated orally.

No need to use Chinese Remainder Theorem. This is the merit of the congruence under consideration.

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