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FUZZY GROUP AND FUZZY SUBGROUP

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ABSTRACT –in this paper we investigate the properties of fuzzy group and fuzzy sub groups and their types. also introduces the notion of a characteristic fuzzy sub group of fuzzy group. we define the algebraic structure of fuzzy sub group and some related properties are investigated. the purpose of this study is to implement the fuzzy set theory and group theory in fuzzy sub group.

KEYWORDS –fuzzy set, fuzzy group, fuzzy subgroup, fuzzy abelian group, fuzzy normal subgroup, left and right coset of fuzzy group.

RESEARCH OBJECTIVES - fuzzy mathematics is a branch of mathematics that is related to fuzzy logic. it started in 1965 after the publication of lotfi asker zadeh's seminal work fuzzy set.

before knowing fuzzy group we should know the fuzzy set. fuzzy group is an extension of theories of fuzzy sets. in this paper we define a new algebraic structureof fuzzy group and fuzzy subgroups and study some of theirrelated properties.

HYPOTHESIS –a hypothesis is an unproven statement which is supported by all the available data and by much weaker result. it is a sequential process of research. it is an indicator of the work to be done for research. it describes the theoretical information in relation to how the research study will work. the hypothesis is a formal suggestion that express the outline of the study's proposal. since zadeh introduced the concept of a fuzzy

set in 1965, various algebraic structures have been fuzzified. as a result the theory of fuzzy group was developed. in 1971 rosenfield introduced the notion of a fuzzy subgroup and thus initiated study of fuzzy group. in this paper we preset the further investigation into properties of fuzzy groups and fuzzy subgroups and also fuzzy normal subgroups.

RESEARCH METHOD – a research design is the set of methods and procedures used in collecting and analyzing measures of variables specified in the problem research. a research design is a frame work that has been created to find answers to research questions. there are numerous types of research design that are appropriate for the different types of research projects. the choice of which design to apply depends on the problems posed by the research aim.

each type of research design has a range of research methods that are commonly used to collect and analyses the type of data, which is generated by the investigations. here is a list of some of more common research design, which is used to find expected outcome of my proposed work.

1. **historical –** this aim at systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusion about past events. it uses primary historical data, such as archaeological remains as well as documentary sources of the past, it is usually necessary to carry out tests in order to check the authenticity of these sources.
2. **descriptive –** this design relies on observation as a means of collecting data. it attempts to examine situations in order to establish. observation can take many forms depending on the type of information sought, people can be interviewed questionnaires distributed.

3. **correlation –** this design is used to examine a relationship between two concepts. there are two broad classifications of relational statements and association between two concepts.
4. **comparative –** this design is used to compare past and present or different parallel situations. it can look at situation at different scales, macro (international, national) or micro (community, individual).

ANALYSIS –fuzzy mathematics is a branch of mathematics that is related to fuzzy logic. it started in 1965 after the publication of lotfi asker zadeh's seminal work fuzzy setbefore knowing fuzzy group we should know the fuzzy set.

fuzzy set-

in this section we shall start to introduce the concept about fuzzy set on the basis definition with some example-

DEFINITION-let a is a non-empty set and the function m: a→ [01] is called fuzzy set in a example- let x be real number r, then the function m: r→ [01] such that

$$m(x) = \begin{bmatrix} 1 - 1/x & if\ x > 1 \\ 0 & if\ x \leq 1 \end{bmatrix}$$

is fuzzy set.

remark- if we want to know the difference between the fuzzy sets and ordinary sets we observe the when a is a set in ordinary sense of the term, so it membership function can take only two values 0 and 1 with a characteristic function

$$a(x) = \begin{bmatrix} 1 & if x \in A \\ 0 & if\ x \notin A \end{bmatrix}$$

then a(x) ∈ {0,1} for all x∈g while if a is a fuzzy set in g then 0≤ a(x)≤1 for all x ∈ g thus the ordinary sets became a special case of fuzzy sets .

fuzzy group –

definition [1] – let g be any group . a mapping : g →[0 1] is a fuzzy group if---

(f g1) μ (x y) ≥ min { μ (x) , μ(y) }

(f g2) μ (x^{-1}) = μ (x) for all x , y ∈ g

definition [2] – a q-fuzzy set a is called q-fuzzy group of g if ---

(q f g1) a (x y, q) = a (x q)

(q f g2) a (x^{-1} , q) = a (x q)

(q f g3) a (e , q) = 1 for all x , y ∈ g and q ∈ q

--- category of fuzzy groups –-

fuzzy abelian group – the fuzzy group (x,) is called a fuzzy abelian group if x is abelian . moreover in this case the binary operation is denoted by" + ",". " thus (x,)is a fuzzy abelian group if..

(1) x is anabelian group.

(2) μ (x + y) ≥ min { μ (x) , μ (y) }

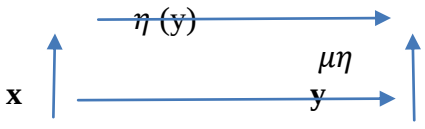
(3) μ (- x) = μ (x)

for all x∈ x

fuzzy group homomorphism-- let(x,μ) and (y, η) be a fuzzy group. then a fuzzy group homomorphism from (x , μ) into (y , η) is a pair (f , α) where f : x → y is a group homomorphism and α : μ (x)→η (y) is a function such that α μ = η f . frequently (f,α): (x,) → (y , η) is a fuzzy group homomorphism or fuzzy morphism if f : x → y is a homomorphism of groups and the following diagram commutes ---

α

μ(x)

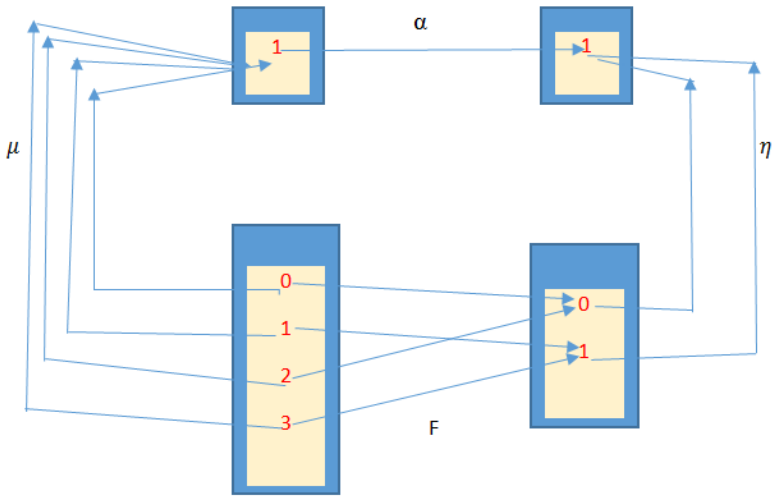


f

example- consider the following example the fuzzy groups (z_4, μ) and (z_2, η) where $\mu(x) = 1$ for all $x \in z_4$ and $\eta(y) = 1$ for all $y \in z_2$

define $f : z_4 \rightarrow z_2$ by $f(0) = f(2) = 0$ and $f(1) = f(3) = 1$ then f is a group homomorphism from $z_4 \rightarrow z_2$

define $\alpha : \mu(z_4) \rightarrow \eta(z_2)$ as $\alpha(\mu(x)) = 1$ for all $x \in z_4$ then α is a bijection and $(f, \alpha) : (z_4, \mu) \rightarrow (z_2, \eta)$ is a fuzzy group homomorphism .



fuzzy subgroup—let g be a group and μ a fuzzy sub set of g . then μ is called a fuzzy subgroup of g if—

(1) $\mu(x\,y) \geq \min\{\mu(x), \mu(y)\}$

(2) $\mu(x^{-1}) = \mu(x)$

(3) μ is called a fuzzy normal subgroup if $\mu(x\,y) = \mu(y\,z)$

for all x and y in g .

properties of (λ, μ) - fuzzy subgroups – in this topic , g stands for a group with identity and $0 \leq \lambda < \mu \leq 1$ by a fuzzy set of g .we mean a mapping from g to the closed unit interval [0 , 1] . in this section we present some basic properties of (λ, μ) fuzzy subgroups.

definition - let a be a fuzzy subset of g then a is called a fuzzy subgroup of g, if for all x, y ∈g.

(1) a (x y) ≥ a(x) ∧ a (y)

(2) a (x^{-1}) ≥ a(x)

properties of (λ , μ) - fuzzy normal subgroups – let a be a fuzzy subgroup of g , a is called a fuzzy normal subgroup of g if for all x y ∈ **g** .

a (x y x^{-1}) ≥ a (y)

properties of left coset and right coset of a -- let a be a (λ, μ) fuzzy sub group of g and a ∈ g define fuzzy subsets a ∘ a and a∘ a of g is that

(a∘ a) (x) = (a (a^{-1} x)∨ λ) ∧ μ for all x ∈ g

(a ∘a) (x) = (a (x a^{-1}) ∨ λ) ∧ μ for all x ∈ g

a∘ a and a∘ a will be called a left coset and right coset of a respectively .

CONCLUSION – in this paper we conclude that the concept of normal fuzzy subgroup and proved some properties of this new concept. we give the basic properties of fuzzy group and fuzzy subgroups. we present a further investigation into properties of fuzzy subgroup, fuzzy normal subgroup and fuzzy left and right coset. certainly some other topics of fuzzy subgroups such as the characterization of fuzzy subgroup and operations of fuzzy subgroups are still open for the research in the further .

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