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VARIETIES OF DETOUR DOMINATION NUMBER OF GRAPHS

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ABSTRACT:

Let $G = (V, E)$ be a graph. A u- v a detour is a longest u- v path. A subset D⊆V is called a detour set of G if every vertex in V-D lie in a detour joining the vertices of D. A subset D⊆V which is both a detour set and dominating set is called a detour dominating set of G and the cardinality of a minimum detour dominating set is called the detour domination number of G. In this paper, we introduce the concept of different detour domination numbers and find the same for some simple and special graphs.

KEYWORDS: Domination, Detour Domination.

AMS Subject Classification: 05C78.

1. INTRODUCTION

Consider finite graphs without loops and multiple edges. For any graph G, the set of vertices is denoted by $V(G)$ and the edge set by $E(G)$. The order and size of G are denoted by p and q respectively. Consider connected graphs with atleast two vertices. For basic definitions and terminologies, we refer [1,7,8]. For vertices u and v in a connected graph G, the detour distance $D(u, v)$ is the length of a longest u- v path in G. A u- v path of length $D(u, v)$ is called a u- v detour. These concepts were studied by Chartrand et al. [2,3]. A vertex x is said to lie on a u- v detour P if x is a vertex of a u- v detour path P including the verticles u and v. A set $S \subseteq V$ is called a detour set if every vertex v in G lies on a detour joining a pair of vertices of S. The detour number $dn(G)$ is called a minimum order of a detour set and any detour set of order $dn(G)$ is called a minimum detour set of G. These concepts were studied by Chartrand [4]. A set $S \subseteq V(G)$ is called a dominating set of G if every vertex in $V(G) - S$ is adjacent to some vertex in S. The domination number $\gamma(G)$ of G is the minimum order of its dominating sets and any dominating set of order $\gamma(G)$ is called a γ-set of G. A detour domianating set is a subset S of $V(G)$ which is both a dominating and a detour set of G. A detour dominating set is said to be minimal detour dominating set of G if no proper subset of S is a detour dominating set of G. A detour dominating set S is said to be minimum detour dominating set of G if there exists no detour dominating set S' such that $|S'| < |S|$. The smallest cardinality of a detour dominating set of G is called the detour domination number of G. It is denoted by $\gamma_d(G)$. Any detour dominating set S of G of cardinality $\gamma_d(G)$ is called a γ_d-set of G.A vertex and an edge are said to cover each other if they are incident. A set of vertices which covers all the edges of a graph G is called a vertex cover for G. The smallest number of vertices in any vertex cover for G is called its vertex covering number and is denoted by $\alpha_0(G)$ or α_0. A set S of vertices in a graph G is independent if no two of its vertices are adjacent in G. The largest number of vertices in such a set is called the vertex independence number of G and is denoted by $\beta_0(G)$ orβ_0. If G is a graph with p vertices, then $\alpha_0(G) + \beta_0(G = p$.A split (G, D)-set S of a graph G is said to be a split (G, D)-set of G if the subgraph induced by $V - S$ is disconnected. A (G, D)-set S of a graph G is said to be a strong split (G, D)-set of G if the subgraph induced by $V - S$ is totally disconnected. Split and Strong split (G, D)-set were introduced in [9]. A dominating set S of a graph G which is also independent is called an independent dominating set of G. The minimum cardinality of all independent dominating sets of G is called its independent domination number and is denoted by $i(G)$. The following theorems are from [3].

1.1 Theorem: Every end vertex of a detour graph G belongs to every detour set of G. Also, if the set of all end vertices of G is a detour set, then S is the unique detour basis for G.

1.2 Theorem: If G is a detour graph of order $p > 3$ such that $\{u, v\}$ is a detour basis of G, then u and v are not adjacent.

1.3 Theorem: If T is a tree with k end vertices, then $dn(T) = k$.

The following theorems are from [5].

1.4 Theorem: K_P is a detour dominating graph and $\gamma_d(K_P) = 2$ for $p \geq 3$.

1.5 Theorem: $\gamma_d(K_{1,n}) = $ n.

1.6 Theorem: $\gamma_d(P_n) = \begin{cases} \left\lceil \frac{n-4}{3} \right\rceil + 2 & if\ n \geq 5 \\ 2 & if\ n = 2, 3\ or\ 4 \end{cases}$

1.7 Theorem: For $n \geq 5$, $\gamma_d(C_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

2. INDEPENDENT DETOUR DOMINATION NUMBER OF GRAPHS

2.1 Definition: A (γ, d)-set S of G is said to be an independent detour dominating set of G if the subgraph induced by S is independent.

2.2 Remark: All graphs do not possess independent (γ, d)-sets. For example complete graph have the vertex set $V(G)$ as the unique (γ, d) -sets. But they are not independent and so complete graphs have no independent detour dominating sets.

2.3 Definition: Let ζ denote the collection of all graphs having atleast one independent detour dominating set. Let G ∈ ζ. Then, the minimum cardinality among all independent detour dominating sets of G is called the independent detour domination number of G. It is denoted by $I\gamma_d(G)$. An independent (γ, d)-set of cardinality $I\gamma_d(G)$ is called an $I\gamma_d$-set of G.

2.4 Example: Consider the graph G in Figure 2.1

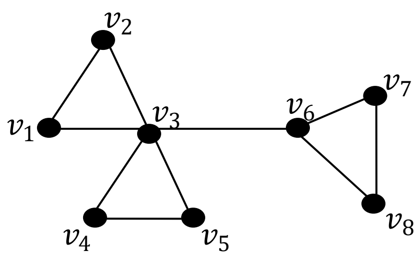

Figure 2.1

$S = \{v_1, v_5, v_8\}$ is a minimum(γ, d)-set of G. Therefore, $\gamma_d(G) = 3$. Further,S is also a minimum independent detour dominating set of G and hence $I\gamma_d(G) = 3$.

2.5 Example: Consider the graph in Figure 4.2

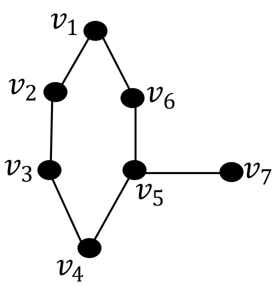

Figure 2.2

$S = \{v_2, v_5, v_7\}$ is the minimum (γ, d)-set of G. Therefore, $\gamma_d(G) = 3$. But, the subgraph induced by S is not an independent whereas $\{v_1, v_4, v_7\}$, $\{v_1, v_3, v_7\}$ are independent and $I\gamma_d(G) = 3$.

2.6 Observation: Let G be a connected graph. Then,

1. C_3, C_5 and C_7 has no independent detour dominating set.
2. Every independent detour dominating set is a detour dominating set of G. Therefore, $2 \leq \gamma_d(G) \leq I\gamma_d(G) \leq p$.
3. If S is a minimum independent detour dominating set of G, then $V - S$ is a dominating set of G.

2.7 Proposition: Let G be a connected graph with p vertices. Let $G \in \zeta$. Then, $I\gamma_d(G) \leq p - \gamma(G)$.

Proof: Let S be a minimum independent detour dominating set of G. By Observation 2.6, $\gamma(G) \leq |V - S|$. Therefore, $\gamma(G) \leq |V| - |S| = p - I\gamma_d(G)$. Hence, $I\gamma_d(G) \leq p - \gamma(G)$.

2.8 Proposition: Let $G \in \zeta$ and let S be an independent detour dominating set of G. If $\delta(G) \geq k$, then $V - S$ is a k −dominating set of G. Further, $I\gamma_d(G) \leq p - \gamma_k(G)$.

Proof: Let $v \in S$. Then, S is independent and $\delta(G) \geq k$ imply that v is adjacent to atleast k vertices of $V - S$. Therefore, $V - S$ is a k −dominating set of G and $\gamma_k(G) \leq |V - S| = |V| - |S| \leq p - I\gamma_d(G)$. Therefore, $I\gamma_d(G) \leq p - \gamma_k(G)$.

2.9 Remark: Let G be a connected graph with $p(\geq 3)$ vertices. Then, $I\gamma_d(G) \leq \beta_0(G) = p - \alpha_0(G)$.

2.10 Proposition: Let $G \in \zeta$ be a connected graph with $p(\geq 3)$ vertices. Then, $I\gamma_d(G) = 2$ if and only if $\gamma_d(G) = 2$.

Proof: Suppose $I\gamma_d(G) = 2$. Then, by Observation 2.6, $2 \leq \gamma_d(G) \leq I\gamma_d(G) = 2$. Therefore, $\gamma_d(G) = 2$.

Conversely, suppose $\gamma_d(G) = 2$. Let $S = \{u, v\}$ be a minimum detour dominating set of G. Since $p \geq 3$, $|V - S| \neq \emptyset$. Further, by Theorem 1.2, u and v are not adjacent and hence $d(u, v) \geq 2$. Then, $\{u, v\}$ is an independent (γ, d)-set of G and $I\gamma_d(G) \leq 2$. By Observation 4.6, $I\gamma_d(G) = 2$.

2.11 Proposition: For $n \geq 3$, $I\gamma_d(P_n) = \gamma_d(P_n)$.

Proof: Let $n \geq 3$ and $P_n = (v_1, v_2, \ldots, v_n)$. If $n \equiv 0 \ (mod\ 3)$ or $n \equiv 1 \ (mod\ 3)$ or

$n \equiv 2\ (mod\ 3)$, then $S = \{v_1, v_4, \dots, v_{n-2}, v_n\}$ or $S = \{v_1, v_4, \dots, v_{n-3}, v_n\}$ or $S = \{v_1, v_4, \dots, v_{n-2}, v_n\}$ is a minimum (γ, d)-set of P_n. Also, S is independent. Therefore, $I\gamma_d(P_n) \leq \gamma_d(P_n)$. Hence, by Observation 2.6, $I\gamma_d(P_n) = \gamma_d(P_n)$.

2.12 Proposition: For the cycle,$(n \geq 4\ and\ n \neq 5)$, $I\gamma_d(C_n) = \gamma_d(C_n)$.

Proof: Obviously, $I\gamma_d(C_4) = \gamma_d(C_4) = 2$.

Let $n > 5$ and let $C_n = (v_1, v_2, \dots, v_n, v_1)$. If $n \equiv 0\ (mod\ 3)$or $n \equiv 1\ (mod\ 3)$ or $n \equiv 2\ (mod\ 3)$, then $S = \{v_1, v_4, \dots, v_{n-2}\}$ or $S = \{v_1, v_3, v_6 \dots, v_{n-4}, v_{n-2}\}$ or $S = \{v_1, v_4, \dots, v_{n-4}, v_{n-1}\}$ is a minimum (γ, d)-set of $C_n$and also, S is independent. Therefore, $I\gamma_d(C_n) \leq \gamma_d(C_n)$. Hence, by Observation 2.6, $I\gamma_d(C_n) \geq \gamma_d(C_n)$. Therefore, $I\gamma_d(C_n) = \gamma_d(C_n)$.

2.13 Proposition: For $n > 5$, $I\gamma_d(\mathrm{W}_{1,n}) = i(C_n)$ where $i(C_n)$ denotes the independent domination number of G.

Proof. Let $n > 5$ Then, any independent dominating set of the outer cycle is also an independent detour dominating set of $\mathrm{W}_{1,n}$and vice versa.

Therefore, $I\gamma_d(\mathrm{W}_{1,n}) = i(C_n)$.

2.14 Proposition: Let $m, n \geq 2$. Then, $I\gamma_d(\mathrm{K}_{m,n}) = \min\{m, n\}$.

Proof: Let W, T be the partition of $V(K_{m,n})$ with $|W| = m$ and $|T| = n$. Let S be an independent detour dominating set of $K_{m,n}$. Then, either $S = W$ or $S = T$. For, if S is a proper subset of W or T, then the vertices of $W - S$ or $T - S$ are not dominated by any vertex of S. Hence, $I\gamma_d(\mathrm{K}_{m,n}) = \min\{|W||T|\} = \min\{m, n\}$.

2.15 Proposition: Let G be a connected graph on p vertices. Then, $G^+ \in \zeta$ and $I\gamma_d(G^+) = p$, where $G^+$is the graph obtained from G by attaching an end vertex to each vertex of G.

Proof: Let $V(G) = (v_1, v_2, \dots, v_p)$ and $w_1, w_2, \dots, w_p$be the end vertices attached to $v_1, v_2, \dots, v_p$respectively in G^+. Then, $S = \{w_1, w_2, \dots, w_p\}$ is the unique minimum independent detour dominating set of G^+ and so $I\gamma_d(G^+) = p$.

3. SPLIT DETOUR DOMINATION NUMBER OF GRAPHS

3.1 Definition: A detour dominating set S of a graph G is said to be a split detour dominating set of G if the subgraph induced by $V - S$ is disconnected.

3.2 Example: For the Figure 3.1, $\{v_1, \mathrm{v}_4\}$ is a minimum detour dominating set of G. It is also a minimum split detour dominating set of G. Therefore, $\gamma^s{}_d(\mathrm{G}) = \gamma_d(\mathrm{G}) = 2$.

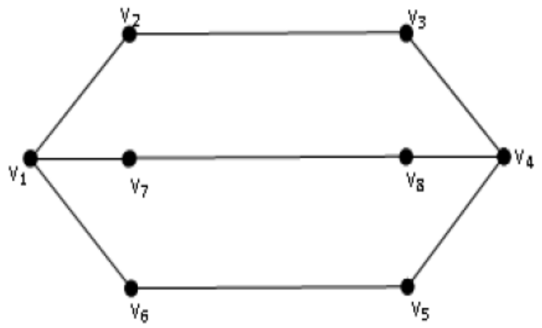

Figure 3.1

3.3 Remark: In general, all graphs need not have split detour dominating sets. For example, K_n has no split detour dominating set.

Let ζ' denote the collection of all graphs having atleast one split detour dominating set.

3.4 Definition: Let $G \in \zeta'$. The minimum cardinality of all split detour dominating set of G is called the split detour domination number of G. It is denoted by $\gamma^s{}_d$(G). A split detour dominating set of minimum cardinality $\gamma^s{}_d$(G) is called $\gamma^s{}_d$- set of G.

3.5 Observation:

1. Every split detour dominating set is a detour dominating set of G. Therefore, for $p \geq 2$, $\gamma^s{}_d$(G) $\geq \gamma_d$(G) $\geq max\{\gamma(G), dn(G)\}$.

2. Every split detour dominating set is a split dominating set of G. Hence, $\gamma^s{}_d(G) \geq \gamma_s(G)$.

3.6 Proposition: Let $G \in \zeta'$. Then, a detour dominating set S of G is a split detour dominating set of G if and only if there exists two vertices $w_1, w_2 \in V - S$ such that every $w_1 - w_2$ path contains a vertex of S.

Proof: Suppose S is a split detour dominating set of G. Then, $V - S$ is not connected and has at least two components. Choose $w_1, w_2 \in V - S$ such that they are in two different components of $V - S$. Therefore, every $w_1 - w_2$ path contains a vertex of S. Converse is obvious.

3.7 Proposition: For $n \geq 6$, $\gamma^s{}_d(P_n) = \gamma_d\ (P_n)$.

Proof: For $n \geq 6$, every minimum detour dominating set S of P_n, $V - S$ is disconnected. Therefore, every detour dominating set of P_n is a split detour dominating set of P_n and $\gamma^s{}_d(P_n) \leq \gamma_d\ (P_n)$. By Observation 2.5, $\gamma^s{}_d(P_n) = \gamma_d\ (P_n)$.

3.8 Proposition: For $n \geq 6$, $\gamma^s{}_d(C_n) = \gamma_d\ (C_n) = \left\lceil \frac{n}{3} \right\rceil$.

Proof: For $n \geq 6$, every minimum detour dominating set S of C_n, $V - S$ is disconnected. Therefore, every detour dominating set of C_n is a split detour dominating set of C_n and so $\gamma^s{}_d(C_n) \leq \gamma_d\,(C_n)$. Therefore proceeding as in 3.7, $\gamma^s{}_d(C_n) = \gamma_d\,(C_n) = \left\lceil \frac{n}{3} \right\rceil$.

3.9 Proposition: For the complete bipartite graph$K_{m,n}$, $m, n \geq 2$, then $\gamma^s{}_d\left(K_{m,n}\right) = min\{m, n\}$.

Proof: Let U, W be the partition of $V(K_{m,n})$ with $|U| = m$ and $|W| = n$. Clearly, U and W are split detour dominating sets of $K_{m,n}$. Let S be a split detour dominating set of $K_{m,n}$. Then, S is not a proper subset of U or W, otherwise $V - S$ is connected. If U or W is a proper subset of S, then S is not a minimal split detour dominating set of G. Therefore, either U & W are the only minimal split detour dominating set of G. Hence, $\gamma^s{}_d\left(K_{m,n}\right) = min\{m, n\}$.

3.10 Proposition: Let G be a connected graph on p vertices. Then, if $G \in \zeta'$then $G^+ \in \zeta'$ and $\gamma^s{}_d(G^+) \leq p + \gamma^s{}_d(G)$.

Proof: Let $V(G) = (v_1, v_2, \ldots, v_p)$ and $w_1, w_2, \ldots, w_p$ be the end vertices attached to $v_1, v_2, \ldots, v_p$respectively in G^+. If D is a minimum split detour dominating set of G, then $S = \{w_1, w_2, \ldots, w_p\} \cup D$ is a split detour dominating set of G^+ and so $\gamma^s{}_d(G^+) \leq p + |S| = p + \gamma^s{}_d(G)$.

4. STRONG SPLIT DETOUR DOMINATION NUMBER OF GRAPHS

4.1 Definition: A detour dominating set S of a graph G is said to be a strong split detour dominating set of G if the subgraph induced by $V - S$ is totally disconnected.

4.2 Example:

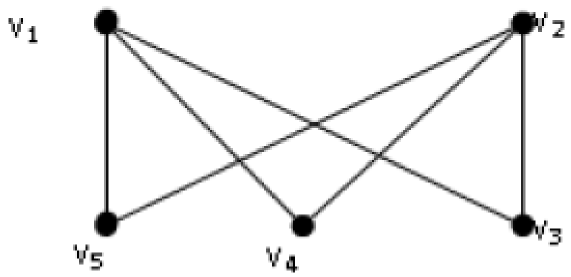

Figure 4.1

Here, $S = \{v_1, v_2\}$ is a strong split detour dominating set of G.

4.3 Observation:

1. For $n > 3$, K_n has no strong split detour dominating set.
2. All graphs need not have a strong split detour dominating set.

Let ζ'' denote the collection of all graphs having atleast one strong split detour dominating set.

4.4 Definition: Let $G \in \zeta''$. Then, the minimum cardinality of all strong split detour dominating sets of G is called the strong split detour domination number of G and is denoted as $\gamma^{ss}{}_{d}$(G). A strong split detour dominating set of minimum cardinality $\gamma^{ss}{}_{d}$(G) is called $\gamma^{ss}{}_{d}$-set of G.

4.5 Observation: Let $G \in \zeta''$.

1. Every strong split detour dominating set is a split detour dominating set of G and a detour dominating set of G. Therefore, $\gamma^{ss}{}_{d}(G) \geq \gamma^{s}{}_{d}(G) \geq \gamma_{d}(G) \geq max\ \{\gamma(G), dn\ (G)\}$.

2. Every strong split detour dominating set is a strong split dominating set of G and a split dominating set of G. Hence, $\gamma^{ss}{}_{d}(G) \geq \gamma_{ss}\ (G) \geq \gamma_{s}(G)$.

3. $2 \leq \gamma_{d}\ (G) \leq \gamma^{s}{}_{d}(G) \leq \gamma^{ss}{}_{d}(G) \leq p$.

4.6 Proposition: Let $G \in \zeta''$. Then, $\gamma^{ss}{}_{d}(G) \geq \alpha_0(G)$.

Proof: Let S be a $\gamma^{ss}{}_{d}-$ set of G.Then, $V - S$ is totally disconnected and hence independent. Therefore, $|V - S| \leq \beta_0(G)$. That is, $p - \gamma^{ss}{}_{d} \leq \beta_0(G) = p - \alpha_0(G)$. Hence, $\gamma^{ss}{}_{d}(G) \geq \alpha_0(G)$.

4.7 Proposition: Let $G \in \zeta''$ and let S be a strong split detour dominating set of G. Then, every $w \in V - S$ lies in a $u - v$ detour of length 2 for some $u, v \in S$.

Proof: Let S be a strong split detour dominating set of G and $w \in V - S$. Since w is adjacent to no vertex of $V - S$ and S is a detour set of G, w lies in a $u - v$ detour P for some $u, v \in S$.

4.8 Corollary: For every $G \in \zeta''$, $\gamma_2\ (G) \leq \gamma^{ss}{}_{d}(G)$.

Proof: Let S be any strong split detour dominating set of G. By Proposition 4.7, every vertex in $V - S$ is adjacent to atleast two vertices of S. Therefore, S is a 2-dominating set of G. In general, every strong split detour dominating set of G is a 2-dominating set of G and hence $\gamma_2\ (G) \leq \gamma^{ss}{}_{d}(G)$.

4.9 Proposition: Let $G \in \zeta''$. Then, a detour dominating set S of G is a strong split detour dominating set of G if and only if for $w_1, w_2 \in V - S$, every $w_1 - w_2$ path contains a vertex of S.

Proof: Suppose S is a strong split detour dominating set of G. Then, $V - S$ is an independent set. Hence, every path joining two vertices of $V - S$ contains a vertex of S.

Conversely, let S be a detour dominating set of G. If two vertices $u, v \in V - S$ are adjacent, then the edge uv is a $u - v$ path in the subgraph induced by $V - S$ and it contains no vertex of S. This is a contradiction to our assumption and hence $V - S$ is totally disconnected. Hence, S is a strong split detour dominating set of G.

4.10 Proposition: For $n > 3$,

$$\gamma^{ss}{}_{d}(P_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if \quad n \; is \quad odd \\ \left\lceil \frac{n}{2} \right\rceil + 1 & \text{Otherwise} \end{cases}$$

Proof: Let $n > 3$ and $P_n = (v_1, v_2, , \dots, v_n)$.

Case-1: n is odd. Let $S = \{v_1, v_3, \dots, v_n\}$. If $w = v_i \in V - S$, then w lies in a $v_1 - v_n$ detour. Further, it is dominated by both v_{i-1} and v_{i+1}.Therefore, S is a detour dominating set of P_n. By construction of S, no two vertices of $V - S$ are adjacent. Therefore, S is a strong split detour dominating set of P_n and so,

$\gamma^{ss}{}_{d}(P_n) \leq |S| = \left\lceil \frac{n}{2} \right\rceil$ ……………(1)

Further, as every detour dominating set contains the end vertices of the path, the maximum cardinality of $V - S$, where S is a strong split detour dominating set of P_n is $\left\lfloor \frac{n}{2} \right\rfloor$.

Therefore, $|V - S| \leq \lfloor \frac{n}{2} \rfloor$. That is, $|S| \geq |V| - \left\lfloor \frac{n}{2} \right\rfloor = \left\lceil \frac{n}{2} \right\rceil$, as n is odd.

Hence, $\gamma^{ss}{}_{d}(P_n) \geq \left\lceil \frac{n}{2} \right\rceil$ …………(2)

From, (1) and (2) $\gamma^{ss}{}_{d}(P_n) = \left\lceil \frac{n}{2} \right\rceil$.

Case-2: n is even. Let $S = \{v_1, v_3, \dots, v_{n-1}, v_n\}$. Since $v_1, v_n \in S$, if $w = v_i \in V - S$, then w lies in a $v_1 - v_n$ detour. Further, it is dominated by both v_{i-1} *and* v_{i+1}. Therefore, S is a detour dominating set of P_n. By the construction of S, no two vertices of $V - S$ are adjacent. Therefore, S is a strong split detour dominating set of P_n and

$\gamma^{ss}{}_{d}(P_n) \leq |S| = \left\lceil \frac{n}{2} \right\rceil + 1$…………. (3)

Further, as every detour dominating set contains the end vertices of the path, the maximum cardinality for $V - S$, where S is a strong split detour dominating set of P_n,is $\left\lceil \frac{n}{2} \right\rceil - 1$.

Therefore, $|V - S| \leq \left\lceil \frac{n}{2} \right\rceil - 1$. That is, $|S| \geq \left\lceil \frac{n}{2} \right\rceil + 1$.

Then, $\gamma^{ss}{}_{d}(P_n) \geq |S| = \left\lceil \frac{n}{2} \right\rceil + 1$. ………..(4)

Hence, from (3) and (4), if n is even, then $\gamma^{ss}{}_{d}(P_n) = \left\lceil \frac{n}{2} \right\rceil + 1$.

4.11 Theorem: For $n > 3$, $\gamma^{ss}{}_d(C_n) = \left\lceil\frac{n}{2}\right\rceil$.

Proof: Let $n > 3$ and $C_n = (v_1, v_2, \ldots, v_n, v_1)$.

Case-1: n is odd. Let $S = \{v_1, v_3, \ldots, v_n\}$ is a strong split dominating set of C_n. Every $w \in C_n$ lie in a $v_1 - v_n$ detour. Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is a detour dominating set of C_n. By construction of S, no two vertices of $V - S$ are adjacent. Therefore, S is a strong split detour dominating set of C_n and so,

$\gamma^{ss}{}_d(C_n) \leq |S| = \left\lceil\frac{n}{2}\right\rceil$ ....(1)

Further, if S is a strong split detour dominating set of C_n, then $V - S$ is independent and the maximum cardinality for $V - S$ is $\left\lfloor\frac{n}{2}\right\rfloor$. Therefore, $\gamma^{ss}{}_d(C_n) \geq \left\lceil\frac{n}{2}\right\rceil$.............(2)

From (1) and (2), if n is odd, then $\gamma^{ss}{}_d(C_n) = \left\lceil\frac{n}{2}\right\rceil$.

Case-2: n is even.

Clearly, $S = \{v_1, v_3, \ldots, v_n\}$ is a strong split detour dominating set of C_n and so $\gamma^{ss}{}_d(C_n) \leq |S| = \frac{n}{2}$. .... (3)

Further, if S is a strong split detour dominating set of C_n, then $V - S$ is independent and the maximum cardinality for $V - S$ is $\frac{n}{2}$.

Therefore, $\gamma^{ss}{}_d(C_n) \geq \frac{n}{2}$ .... (4)

From (3) and (4), $\gamma^{ss}{}_d(C_n) = \frac{n}{2}$..

Hence, for $n > 3$, $\gamma^{ss}{}_d(C_n) = \left\lceil\frac{n}{2}\right\rceil$.

4.12 Theorem: For $m, n \geq 2$, $\gamma^{ss}{}_d(K_{m,n}) = min\ \{m, n\}$.

Proof: Let U, W be the partition of $V(K_{m,n})$ with $|U| = m$ and $|W| = n$. Obviously, U and W are strong split detour dominating sets of $K_{m,n}$. If S is a strong split detour dominating set, then $V - S$ is independent. Therefore, S does not contain part of U as well as part of W. If U or W is a proper subset of S, then S is not a minimal strong split detour dominating sets of $K_{m,n}$. Therefore, U and W are the only minimal strong split detour dominating sets of $K_{m,n}$. Hence, $\gamma^{ss}{}_d(K_{m,n}) = min\ \{|U|, |W|\} = min\ \{m, n\}$.

5.Conclusion:

In this paper we have analysed the different detour domination numbers of some simple and special graphs. It is interesting to investigate further the detour domination number of many other special classes of graphs that are widely used in other areas of research in graph theory.

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