International Journal of Engineering, Science and Mathematics
Vol. 8 Issue 12, December 2019,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

POSSIBILISTIC APPROACH FOR SELECTION OF CRITICAL PATH IN MULTI OBJECTIVE TRAPEZOIDAL FUZZY ENVIRONMENT

REKHRIDDHIKETANKUMAR*

DR. JAYESH M. DHODIYA**

ABSTRACT

This paper contains possibilistic method based on a linear and exponential membership function and its application to define the critical path in project network. It contains four criteria - cost, duration, quality and risk of the project activities which are to be considered as critical in project organisation. A model is formulated for selection of critical path in trapezoidal fuzzy environment. For find the solution of this multi-criteria project management problem fuzzy programming technique with linear and exponential membership is utilized with alpha level set concept. We have provided numerical illustration to validate working of the proposed approach. To analyze the performance of the proposed approach, we have compared it with closely related fuzzy group multi-criteria decision making method for the critical path selection. Degree of satisfaction is calculated for different values of α levels to validate the applicability of this new approach.

KEYWORDS:

Project management;
α-level of trapezoidal fuzzy number;
Possibilistic approach;
Fuzzy programming;
Critical path.

Author correspondence:

Rekh Riddhi Ketankumar,
Assistant Professor, RNGPIT, Isroli-Afwa,
Bardoli,Dist. Surat,Gujarat , India

* Assistant Professor, RNGPIT, Isroli-Afwa, Bardoli,Dist. Surat ,Gujarat , India.
**Associate Professor, SVNIT, Surat,Gujarat , India.

1. INTRODUCTION

Project management is extremely important theory to recognize and apply to the projects started by nearly all the organizations in recent competitive business environment. How activities of project should be carried out in an effective manner, when resources are restricted, is worked out by project management theory. The application of project management to plan activities and supervise development within definite duration, risk, cost, and performance strategies is highly essential, so as to attain competitive main concerns such as customization and on-time delivery .PERT and CPM are methods of operations research used for planning,scheduling and controlling large and complex projects. Both the methods require to represent the project as Network diagram of activities of the project[33].

To help US Navy's Polaris Nuclear Submarine Missile project involving thousands of activities in the planning and scheduling and for that a research team developed PERT in 1956-58. The aim of the team was to powerfully design and grow the Polaris missile structure. This technique was useful since 1958 for all jobs or projects having an element of uncertainty in the estimation of duration, just like with new types of projects. Such approach has never been taken up before. Critical Path Method (CPM) was developed independently, by E.I. Du Pont Company with Remington Rand Corporation at the same time. The aim behind its development was to provide a technique for control of the maintenance of company's chemical plants. The core objective before initiating any project is to plan all essential activities in an effective method so as to complete it within a definite duration and with minimised cost for completion**.** For scheduling and maintaining complex projects in real-world applications, CPM is the useful project management technique. This technique is helpful to project administrators to caltulate the minimum completion duration and critical activities of the project so as to decide where capitals, material and men power must be focussed more in order to decrease project finishing time [36].

Kelly [27] developed and solved the time-cost trade-off problem by heuristic algorithm and mathematical modelling by assuming a linear relation among cost and duration of an activity. A special parametric linear program for CPM thatcan be effectively solved by network flow methods was developed by the author. The model provides solutions to concerning project budget, labor requirements, procurement and plan restrictions, the results of slowdowns and conveyance problems. Amiri and Golozari[1] developed an algorithm to find the critical path by considering four criterias- cost, duration, quality and risk in fuzzy environment. Chanas, Dubois and Zielin´ski[5] developed two applicable algorithms for defining the paths to the maximum degree of criticality and degree of necessary criticality of a fixed path in fuzzy environment. Dubois, Fargierand Galvagnon [19] presented project analysis by defining interval-valued durations and then extended them to fuzzy intervals .Chanas and Zielin´ski [7] applied Zadeh's principle and developed two approaches of calculation of degree of criticality of the path with fuzzy activity times.Chanas and Zielin´ski[8] considered a project network with activities times as interval and examined criticality concept.Chanas and Zielinski [9] obtained complexity outcomes for projects of estimating the criticality of activities in planar systems with duration time intervals.

Chen SP [13] applied Yager ranking technique and the relative degree of criticality of paths to find critical path in a project network where activity times were *L-R* and *L-L* type trapezoidal fuzzy numers. Chen SP, Hsueh YJ [14] proposed a method based on fuzzy number ranking and LP formulation to calculate critical path in a project network in fuzzy environment with single criteria-time. Cristobal JRS [18] developed PROMETHEE method to calculate the critical path of a network with four criterias cost, duration, safety and quality. Elizabeth and Sujatha[20]developed a ranking based methodology to find critical path with considering activity durations as triangular fuzzy numbers. Lin CT and Ying TC [31] developed a method focused on bid or no-bid decision making with fuzzy numbers and proposed a case study of airplane project in Taiwan. Mahdi andAlreshaid [32]utilized a multi-criteria decision-making process by using the analytical hierarchy process (AHP) for good delivery system for their projects. Chen SP [31]proposed linear programs with possibility level α to find upper and lower bounds of the total project duration at α, considering activity times as fuzzy numbers for a project network and calculated critical path. Concepts of the best critical path, Yager ranking method,and the relative degree of criticality of paths were established. A multi-objective optimal control problem that contains four conflicting objective functions was developed by Azaron[3].In many situations the process time for every activity is generally tough to describe and approximate exactly in a actual situation. Concept of fuzzy set theory and fuzzy numbers is very useful in such situations. Chen and Huang [10] proposed an theoretical technique for determining the criticality in a project network with fuzzy activity durations. Operation times for all activities in a project network were expressed by Triangular fuzzy numbers. Authors developed a novel model by combining PERT technique with fuzzy set theory to define the critical degrees of paths and activities.

FPERT method was developed by Chanasand Kamburowski [6] for approximating a project finishing duration in the condition when activity durations in the project diagram are in terms of fuzzy variables. Authors have proposed two approaches to calculate the degree of fuzzy criticality of path in project netwok. Chen and Chang [12] proposed a fuzzy PERT algorithm to calculate various likely critical paths in a project network, where the duration of each activities was denoted by a fuzzy number.

TOPSIS method was established by Hwang and Yoon [16] .Liang [30]established a unique decision process based on anti-ideal and ideal concepts for ranking order for three sites under trapezoidal fuzzy environment. Ashtiani[2] presented the fuzzy TOPSIS method by considering criterias as interval-values for solution of MCDM problems with unequal weights of criterias, applying fuzzy sets concepts. Chen [15] considered weight of criterias in terms linguistic variables and experessed them into triangular fuzzy numbers and applied the **TOPSIS** method. Benitez[4] presented a fuzzy multi-assignment decision-making technique for calculating new ideas for the amenity worth of three hotels of an top most company in Gran Canaria island via reviews. Authors applied TOPSIS andcalculated common facility performance index for each pair date –hotel of review, built on the idea of the optimality degree. Four criterias such as price, quality, delivery performance and flexibility are important for suppliers selection. Chen and Huang [11] developed a decision-making method for choice of supplier problem in supply chain organisation. Authors applied concept of the TOPSIS to find ranking order of all suppliers. Chen and Tsao [16]extended the TOPSIS for interval-valued fuzzy data.Chuand Lin [17] , suggested a fuzzy TOPSIS model, interval arithmetic of fuzzy numbers were used to define the membership function of each fuzzy weighted rating. To complete the fuzzy TOPSIS model, a ranking technique was utilized to find PIS and NIS. Ekmekcioglu [21] developed revised fuzzy TOPSIS procedure for the choice of suitable removal process and site for municipal solid waste, with the capability to represent imprecise qualitative records and offering all possible results with distinct membership degrees. This approach was superior to existing methods.

Jahanshahloo [23] used concept *of* α-cuts on triangular fuzzy criterias and applied extended TOPSIS to decision-making problems involving fuzzy criteria. Wang and Chang [39]formulated an solution method based on TOPSIS to select optimum starting training aircraft with linguistic terms expressed by triangular fuzzy numbers. The choice of location of plant is highly significant part to minimize cost and maximize the use of resources for organizations. Yong [40] presented a original TOPSIS process for choice of plant location under linguistic conditions. Author utilized ratings of some alternative locations under several criteria, and the weights of different criteria are measured in linguistic relations denoted by fuzzy numbers.Yuan [41] suggested a new modified ranking method with four criterias.Chen SP[13] applied linear programming method and the extension principle for analysis of critical path for a project network with activity durations were fuzzy numbers. Authors calculated relative degree of criticality of paths for analysis of project network.Mehlawat and Gupta [33] developed a fuzzy group decision making process and apply it to define the critical path in a project network. Four criterias cost, duration, quality and risk are measured in linguistic variable and later transformed into triangular fuzzy numbers. Authors expressed a criticality measure in terms of the total performance score of each project path attained by its strength and weakness index scores.Kahraman[24] proposed a fuzzy hierarchical TOPSIS model for the multiple-criteria calculation of the engineering robotic systems. Authors presented application with some sensitivity studies by altering the critical parameters.Kannan[25] proposed a multi-criteria group decision-making (MCGDM) model in fuzzy situation for the selection of third-party reverse logistics providers from among 15 alternatives. Author applied TOPSIS for the analysis. Kaya[26]developed a revised fuzzy TOPSIS approach for the choice of the finest energy technology substitute.Kim [28]suggested an agent-based diffusion model containing of tens of thousands of cooperating mediators that can assist in the analysis of the automobile market. Fuzzy TOPSIS process is utilized to observe the collective behaviour of three buying forces. Kutlu[29]proposed a fuzzy method that allow specialists to practice linguistic variables for determining three risk factors- occurrence, severit and detectability, by applying fuzzy TOPSIS combined with fuzzy ‘analytical hierarchy process’ (AHP).

All the studies of CPM in Trapezoidal as well as triangular fuzzy environment in literature are focused on relative degree of criticality of paths and strongness index as well as weakness index. The purpose of this study is to develop a new method that can find a critical path in trapezoidal fuzzy environment that optimizes all four objective functions with good degree of satisfaction, without finding criticality index of each path.

2. FUZZY MULTI OBJECTIVE CRITICAL PATH PROBLEM FORMULATION (FMOCPP)

The main assumptions and characteristics of the FMOCPP are as follows:

(1) Each path of the project network will be considered.

(2) Dummy activity is considered with all objective values as zero.

(3) The decision making matrix should minimize Time, Cost, Risk and maximize Quality.

(4) Trapezoidal fuzzy numbers are considered for Linguistic variables.

3. FUZZY MULTI OBJECTIVE CRITICAL PATH PROBLEM MODEL

The mathematical formulation of FMOCPP are made by using the following variables, parameters and the indices.

(1) Indices i and j defines path joining node i and j.

(2) E = Set of arcs of the project network , $(i, j) \in E$

(3) Decision variables $x_{ij} = \begin{cases} 1; \ if \ activity \ ij \ lies \ on \ critical \ path \\ 0; otherwise \end{cases}$

4. FORMULATION OF OBJECTIVE FUNCTIONS

The total consumed time, total cost, total quality level and total risk are given as follows:

$$\tilde{z}_1 = \sum_{i,j \in E} \tilde{t}_{ij} \, x_{ij} \quad \tilde{z}_2 = \sum_{i,j \in E} \tilde{c}_{ij} \, x_{ij} \quad \tilde{z}_3 = \sum_{i,j \in E} \tilde{q}_{ij} \, x_{ij} \quad \tilde{z}_4 = \sum_{i,j \in E} \tilde{r}_{ij} \, x_{ij}$$

In this problem, the quality of the linguistic variable are rated as “very low”, “low”, “medium low”,“medium”,“medium high”,“high” and “very high”, which are represented as (0,1,1,1), (0,1,3,5), (1,3,5,7), (3,5,7,9), (5,7,9,11), (7,9,10,12), and (9,9,10,10) respectively. The seven levels represent the quality of project completion, where “very high” and “very low” levels denote the most efficient and least efficient, respectively, that is, a shift from “very high” to “very low” indicates that quality decreases whereas the related fuzzy values increase. Here quality objective functions are convert in minimum form to maintain uniformity of objective functions.

5. MODEL CONSTRAINTS

The constraints of FMOCPP are formulated as follows:

$$\sum_j x_{1j} = 1 \quad (1)$$

$$\sum_j x_{ij} = \sum_k x_{kj} \ , i = 2, 3, \ldots, n-1. \quad (2)$$

$$\sum_k x_{kn} = 1 \quad (3)$$

$$x_{ij} \geq 0, \forall (i, j) \in E \quad (4)$$

6. DECISION PROBLEM

The FMOCPP is now formulated as follows:

(Model -1)

$$(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4) = \left(\sum_{i,j \in E} \tilde{t}_{ij} \, x_{ij} \ , \sum_{i,j \in E} \tilde{c}_{ij} \, x_{ij} \ , \sum_{i,j \in E} \tilde{q}_{ij} \, x_{ij} \ , \sum_{i,j \in E} \tilde{r}_{ij} \, x_{ij} \right)$$

Subject to the constraints (1)-(4).

7. SOME PRELIMINARIES

To find the solution of this fuzzy project management problem some are required which are as follows

7.1 Possibilistic programming approach

Most of the time when we collect real-world problems related data then generally its include some kind of unreliability which are represented using fuzzy numbers because of their nature. Possibilistic distribution is utilized to quantify such kind of fuzzy numbers . Many crucial applications have been used possibilistic programming approach for finding the solution of multi criteria's based fuzzy optimization model with unspecific objective function [37]. Hence in this paper we have utilized possibilistic programming based approach to solve FMOCPP which maintain the uncertainty of the problem in real sense and convert the FMOCPP in crisp MOCPP.

7.2 Trapezoidal possibilistic distribution (TPD)

Trapezoidal possibility distribution [22] is used to represent the trapezoidal uncertain parameter. In particular, for the time coefficient $\tilde{t}_i = (t_i^o, t_i^m, t_i^{\bar{m}}, t_i^p)$, decision maker can create the trapezoidal distribution by using $(t_i^o), ([t_i^m, t_i^{\bar{m}}])$ and (t_i^p) where (t_i^o) and (t_i^p) are themost optimistic value and most pessimistic value respectively (possibility degree = 0), $([t_i^m, t_i^{\bar{m}}])$ is the interval of the most likely value that absolutely belongs to the set of available values (possibility degree = 1).

From figure 1, the time objective function is defined at four well-known points $(t_1^p, 0), (t_1^m, 1), (t_1^{\bar{m}}, 1)$ *and* $(t_1^o = 0)$ and it is minimized by pushing the four positions of trapezoidal possibility distribution. Since the left as vertical coordinates of the points are fixed by 1 or 0, there are only four horizontal coordinates considered.

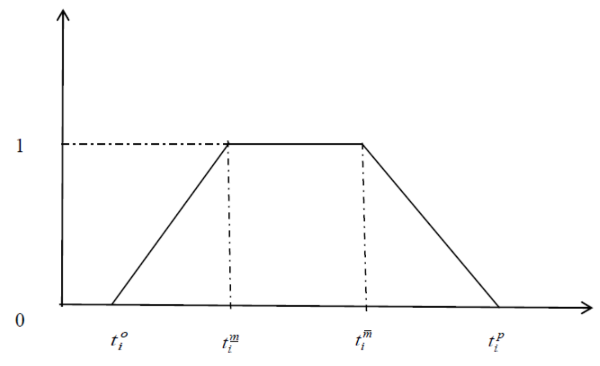

Figure 1 Trapezoidal Possibility distribution of t_i

7.3 Defination of trapezoidal fuzzy

A trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ with the membership function is defined [22] as follows:

$$\mu_{\tilde{a}}(x)=\begin{cases} 0, x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, a_3 \le x \le a_4 \\ 0, a_4 \le x \end{cases}$$

7.4 α - level sets

Several researchers ([34],[37],[38],[42])have used this α-level set concept to find the solutions for fuzzy optimization-related problems. To set up a connection between traditional and fuzzy set theories, an α-level set is the most extremely important theory which was introduced by Zadeh. Largest α-value indicate the greater degree of membership in the initial fuzzy sets with upper and lower bond which is useful a smaller but more optimistic judgment. Generally, α-level indicate the DM confidence with his fuzzy judgement is also named as the confidence level. An interval judgment with a large spared, which point out a high level of pessimism and uncertainty is provided by smallest α-value. We have used this concept in the present study to determine the confidence of the DM with respect to his fuzzy judgment.

7.5 α - cut of trapezoidal fuzzy number

Let $\tilde{a}=(a_1,a_2,a_3,a_4)$ be a trapezoidal fuzzy number. An α -cut for $\tilde{a}$, $\tilde{a}_\alpha$ is computed as:

$((a_2-a_1)\alpha+a_1, a_2, a_3, a_4-(a_4-a_3)\alpha)$.

7.6 Linear Membership function

A linear membership function can be defined as follows.

$$\mu_{z_{ij}}(x)=\begin{cases} 1 & , if\ z_{ij} < z_{ij}^{PIS} \\ 1-\dfrac{z_{ij}-z_{ij}^{PIS}}{z_{ij}^{NIS}-z_{ij}^{PIS}} & , if\ z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS} \\ 0 & , if\ z_{ij} > z_{ij}^{NIS} \end{cases}$$

7.7 Exponential membership function

$$\mu_{z_{ij}}^{E}(x) = \begin{cases} 1 & if \ z_{ij} \le z_{ij}^{PIS} \\ \dfrac{e^{-s\psi_{ij}(x)} - e^{-s}}{1 - e^{-s}}; & if \ z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS} \\ 0 & if \ z_{ij} \ge z_{ij}^{NIS} \end{cases}$$

where $\psi_{ij} = \dfrac{z_{ij} - z_{ij}^{PIS}}{z_{ij}^{NIS} - z_{ij}^{PIS}}$ and s is non-zero shape parameter given by DM that $0 \le \mu_{z_{ij}}(x) \le 1. For\ s > 0\,(s < 0)$, the membership function is strictly concave (convex) in $[z_{ij}^{PIS}, z_{ij}^{NIS}]$. The value of this fuzzy membership function allows us to model the grades of precision in corresponding objective function.

8. FORMULATION OF MULTI OBJECTIVE 0-1 PROGRAMMING MODEL

To convert model 1 into auxiliary multi-objective optimization model, we used Trapezoidal possibilistic distribution (TPD) strategy to treat the imprecise objectives. The Cost, time, risk and quality objective functions are described as

$$\min \tilde{z}_1 = \min(z_1^o, z_1^m, z_1^{\bar{m}}, z_1^p) = \sum_{i,j\in E} \tilde{t}_{ij} x_{ij}$$

$$= \min\left(\sum_{i,j\in E} t_{ij}^0 x_{ij}, \sum_{i,j\in E} t_{ij}^m x_{ij}, \sum_{i,j\in E} t_{ij}^{\bar{m}} x_{ij}, \sum_{i,j\in E} t_{ij}^p x_{ij} \right)$$

where $t_{ij} = (t_{ij}^o, t_{ij}^m, t_{ij}^{\bar{m}}, t_{ij}^p)$, which can be considered as follows:

$$(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}) = \min\left(\sum_{i,j\in E} t_{ij}^o x_{ij}, \sum_{i,j\in E} t_{ij}^m x_{ij}, \sum_{i,j\in E} t_{ij}^{\bar{m}} x_{ij}, \sum_{i,j\in E} t_{ij}^p x_{ij} \right)$$

(5)

Similarly, objective functions for cost, risk and quality criteria are defined as follow.

$$(\min z_{21}, \min z_{22}, \min z_{23}, \min z_{24}) = \min\left(\sum_{i,j\in E} c_{ij}^o x_{ij}, \sum_{i,j\in E} c_{ij}^m x_{ij}, \sum_{i,j\in E} c_{ij}^{\bar{m}} x_{ij}, \sum_{i,j\in E} c_{ij}^p x_{ij} \right)$$

(6)

$$(\min z_{31}, \min z_{32}, \min z_{33}, \min z_{34}) = \min\left(\sum_{i,j\in E} q_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{p}\, x_{ij}\right)$$

(7)

$$(\min z_{41}, \min z_{42}, \min z_{43}, \min z_{44}) = \min\left(\sum_{i,j\in E} r_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{p}\, x_{ij}\right)$$

(8)

Equations (5) - (8) are associated with four different values of trapezoidal fuzzy numbers.

Hence the model becomes

(Model -2)

$$(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}, \min z_{21}, \min z_{22}, \min z_{23}, \min z_{24}, \min z_{31}, \min z_{32},$$
$$\min z_{33}, \min z_{34}, \min z_{41}, \min z_{42}, \min z_{43}, \min z_{44})$$

$$= \left(\begin{array}{l} \sum_{i,j\in E} t_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} t_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} t_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} t_{ij}^{p}\, x_{ij} \\ \sum_{i,j\in E} c_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} c_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} c_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} c_{ij}^{p}\, x_{ij}, \\ \sum_{i,j\in E} q_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} q_{ij}^{p}\, x_{ij}, \\ \sum_{i,j\in E} r_{ij}^{o}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{m}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} r_{ij}^{p}\, x_{ij} \end{array}\right) \quad (9)$$

Subject to the constraints (1)-(4).

Using the α-level set concepts $(0 \le \alpha \le 1)$, each t_{ij} can be stated as.

$$(t_{ij})_{\alpha} = ((t_{ij}^{m} - t_{ij}^{o})\alpha + t_{ij}^{0},\ t_{ij}^{m}, t_{ij}^{\bar{m}},\ t_{ij}^{p} - \alpha(t_{ij}^{p} - t_{ij}^{\bar{m}}))$$

Equation (5) can be written as:

$$(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}) = \left(\sum_{i,j\in E} (t_{ij})_{\alpha}^{o}\, x_{ij}, \sum_{i,j\in E} (t_{ij})_{\alpha}^{m}\, x_{ij}, \sum_{i,j\in E} (t_{ij})_{\alpha}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} (t_{ij})_{\alpha}^{p}\, x_{ij}\right) \quad (10)$$

Similarly, multi-objective optimization problem (MOP) model of cost, risk and quality objective functions are as follows:

$$(\min z_{21}, \min z_{22}, \min z_{23}, \min z_{24}) = \left(\sum_{i,j\in E} (c_{ij})_{\alpha}^{o}\, x_{ij}, \sum_{i,j\in E} (c_{ij})_{\alpha}^{m}\, x_{ij}, \sum_{i,j\in E} (c_{ij})_{\alpha}^{\bar{m}}\, x_{ij}, \sum_{i,j\in E} (c_{ij})_{\alpha}^{p}\, x_{ij}\right)$$

(11)

$$(\min z_{31}, \min z_{32}, \min z_{33}, \min z_{34}) = \left(\sum_{i,j \in E} (q_{ij})_{\alpha}^{o} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{p} x_{ij} \right)$$

(12)

$$(\min z_{41}, \min z_{42}, \min z_{43}, \min z_{44}) = \left(\sum_{i,j \in E} (r_{ij})_{\alpha}^{0} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{p} x_{ij} \right)$$

(13)

8.1 Auxiliary multi-objective 0-1 programming model

To determine the optimistic, most-likely, and pessimistic scenarios by using the α-level set concept, the FMOCPP is converted into a crisp MOCPP also called as an auxiliary multi objective 0–1 programming model which is defined as follows:

(Model -3)

$$\begin{aligned}&(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}, \min z_{21}, \min z_{22}, \min z_{23}, \min z_{24},\\ &\min z_{31}, \min z_{32}, \min z_{33}, \min z_{34} \min z_{41}, \min z_{42}, \min z_{43}, \min z_{44})\end{aligned}$$

$$= \begin{pmatrix} \sum_{i,j \in E} (t_{ij})_{\alpha}^{o} x_{ij}, \sum_{i,j \in E} (t_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (t_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (t_{ij})_{\alpha}^{p} x_{ij}, \\ \sum_{i,j \in E} (c_{ij})_{\alpha}^{o} x_{ij}, \sum_{i,j \in E} (c_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (c_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (c_{ij})_{\alpha}^{p} x_{ij}, \\ \sum_{i,j \in E} (q_{ij})_{\alpha}^{o} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (q_{ij})_{\alpha}^{p} x_{ij} \\ \sum_{i,j \in E} (r_{ij})_{\alpha}^{o} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{m} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{\bar{m}} x_{ij}, \sum_{i,j \in E} (r_{ij})_{\alpha}^{p} x_{ij} \end{pmatrix} \quad (14)$$

Subject to the constraints (1)-(4).

9. FUZZY PROGRAMMING TECHNIQUE-BASED SOLUTION APPROACH TO SOLVE AUXILIARY MODEL OF FMOCPP

For finding the solution of the Model 3 by fuzzy programming techniquefirst this models are solved for single objective function and for each objective function find out the positive ideal solution (PIS) and negative ideal solution (NIS) of the model. Now, by positive ideal solution (PIS) and negative ideal solution (NIS) define a membership function $\mu(Z_k)$for the k^{th}objective function. Here, different membership functions are utilized to find an efficient solution of this multi-objective critical path problem and by using this membership functions the Model 3 is converted into the following models:

Model 3.1:

Max λ,

Subject to the constraints:

$\lambda \le \mu_{z_{ij}}$; $0 \le \lambda \le 1$ (15)

equation (1) to equation (4).

When we utilize fuzzy linear membership function,

$$\mu_{z_{ij}}(x) = \begin{cases} 1, & \text{if } z_{ij} \le z_{ij}^{PIS}, \\ \dfrac{z_{ij}^{NIS} - z_{ij}}{z_{ij}^{NIS} - z_{ij}^{PIS}}, & \text{if } z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS}, \\ 0, & \text{if } z_{ij} \ge z_{ij}^{NIS}, \end{cases} \qquad (16)$$

then model 3.1 structure is as follows:

Model 3.2

Max λ,

Subject to the constraints:

$$\lambda \le \frac{z_{ij}^{NIS} - z_{ij}}{z_{ij}^{NIS} - z_{ij}^{PIS}} \qquad (17)$$

equation (1) to equation (4).

When we utilize exponential membership function,

$$\mu_{z_{ij}}(x) = \begin{cases} 1, & \text{,if } z_{ij} \le z_{ij}^{PIS}, \\ \dfrac{e^{-S\Psi_k(x)} - e^{-S}}{1 - e^{-S}} & \text{,if } z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS}, \\ 0 & \text{,if } z_{ij} \ge z_{ij}^{NIS} \end{cases} \qquad (18)$$

where, $\Psi_{ij}(x) \le \dfrac{z_{ij} - z_{ij}^{PIS}}{z_{ij}^{NIS} - z_{ij}^{PIS}}$, and S is a non-zero parameter, prescribed by the decision maker, then model 4 structure is as follows:

Model 4

Max λ,

Subject to the constraints:

$$\left(e^{-S\Psi_k(y)} - e^{-S}\right) \ge \lambda\left(1 - e^{-S}\right) \text{ where, } \Psi_k(y) \le \frac{Z_k(y) - Z_{ij}^{PIS}}{Z_{ij}^{NIS} - Z_{ij}^{PIS}}, k = 1, 2, ..., n.$$

(19)

with constraints (1) to (4).

10. ALGORITHM:

Input: *Parameters:* $(Z_1, Z_2, ..., Z_m, n)$

Output : ***S****olution of FMOCPP*

Solve FMOCPP $(Z_k \downarrow, X \uparrow)$

begin

read: *problem*

while *problem = FMOCPP do*

for *k=1 to m do*

enter matrix Z_k

end

-| ***find triangular possibilities distribution for each objective function.***

-| ***define the crisp multi-objective critical path problem according to*** $\alpha-$ ***level***

-| ***determine the positive ideal solution(PIS) and negative ideal solution(NIS) for each objective.***

for *k=1 to m do*

$$z_{ij}^{\text{PIS}} = \min (z_i)_{\alpha}^{0}, i, j = 1, 2, 3$$

Subject to constraints (1) to (4),

end

for *k=1 to m do*

$$z_{ij}^{\text{NIS}} = \max (z_i)_{\alpha}^{0}, i, j = 1, 2, 3$$

Subject to constraints (1) to (4),

end

-| ***Define linear or exponential membership function for each objective.***

for *k=1 to m do*

$$\mu_{z_{ij}}(x) = \begin{cases} 1, & \text{if } z_{ij} \le z_{ij}^{PIS}, \\ \dfrac{z_{ij}^{NIS} - z_{ij}}{z_{ij}^{NIS} - z_{ij}^{PIS}}, & \text{if } z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS}, \\ 0, & \text{if } z_{ij} \ge z_{ij}^{NIS}, \end{cases}$$

or

$$\mu_{z_{ij}}^{E}\left(x\right)=\begin{cases}1, & \text{if } \ z_{ij}\le z_{ij}^{\text{PIS}} \\ \dfrac{e^{-S\psi_{ij}(x)}-e^{-S}}{1-e^{-S}}, & \text{if } \ z_{ij}^{\text{PIS}}< z_{ij}< z_{ij}^{\text{NIS}} \\ 0, & \text{if } \ z_{ij}\ge z_{ij}^{\text{NIS}}\end{cases}$$

end

-| ***find single objective optimization model under given constraints from MOP model.***

for*k=1 to m do*

Maximize $=\lambda$,

Subject to:

Constraints (1)-(4) and model 3.2 or model4

$\lambda \ge 0$;

end

|- *find the solution SOP using LINGO software*

11. FLOWCHART

Flowchart of the solution procedure of FMOCPP is shown in Figure 2.

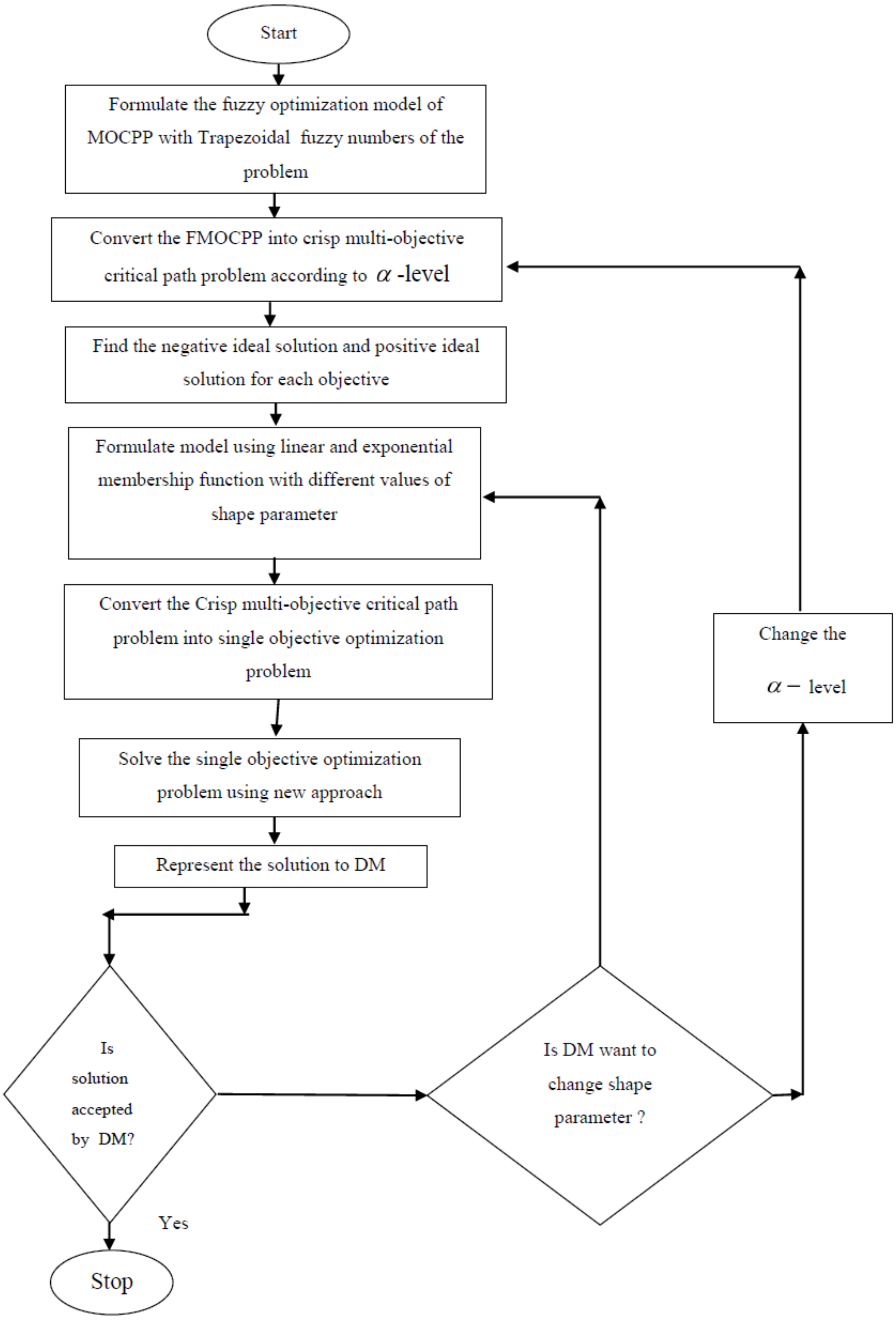

Figure 2 Flow chart

12. NUMERICAL ILLUSTRATION

In this paper, we have considered a large project of [35] constructing a building. Construction of a huge building contains number of important tasks that should be completed in proper order with optimum time, cost, quality and risk. Table 1shows tasks involved in construction of a huge building.

Activity	Description
1-2	Obtain material for beams, Excavate foundations, Obtain Bricks, Obtain wood
1-3	Obtain sanitary fittings,etc
1-4	Obtain electric equipments
2-5	Lay foundations, Brick work, Place roof timbers
3-5	Lay drains
3-6	Plumbing
6-10	Place sanitary fittings
4-6	Plaster
4-8	Electric wiring
8-9	Board fitting
9-10	main connection
5-7	complete roofing, carpentry
7-10	Fit enterer doors, etc

Table 1. Description of activity of the Project

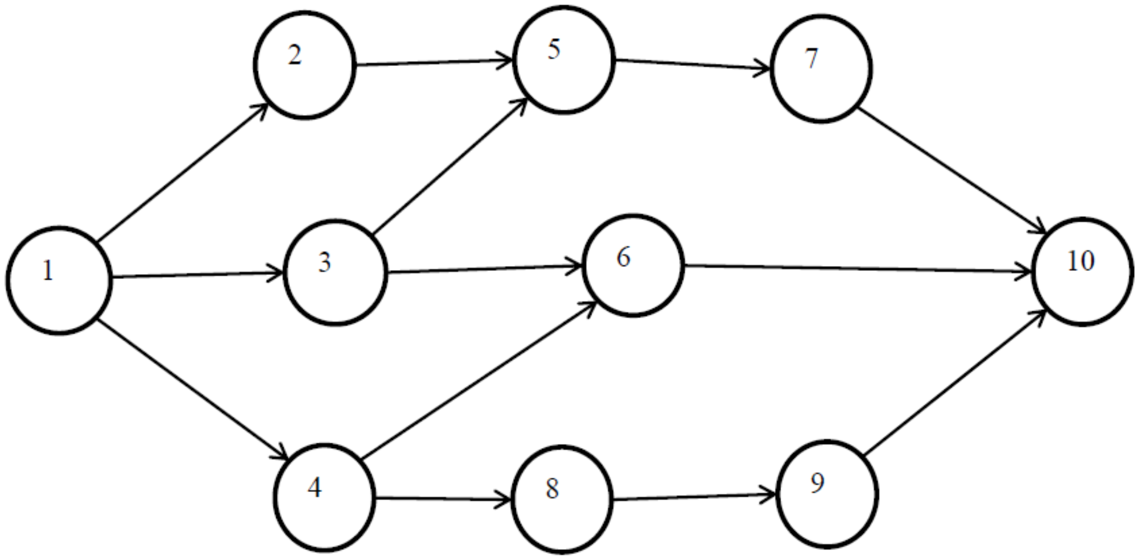

Figure 3. Project Network

Activity	Time	Cost	Risk	Quality
1-2	(4,7,10,12)	(1500,2000,2500,3000)	(5,7,9,11)	(1,2,3,4)
1-3	(3,6,9,12)	(3500,1000,1500,2000)	(1,3,5,7)	(3,4,5,6)
1-4	(2,4,6,8)	(200,700,1200,1700)	(1,3,5,7)	(2,3,4,5)
2-5	(3,5,7,9)	(200,700,1200,1700)	(3,5,7,9)	(3,4,5,6)
3-5	(3,4,5,6)	(1500,2000,2500,3000)	(3,5,7,9)	(3,4,5,6)
4-6	(2,3,4,5)	(5500,6000,6500,7000)	(3,5,7,9)	(2,3,4,5)
8-9	(8,10,12,14)	(1500,2000,2500,3000)	(1,3,5,7)	(2,3,4,5)
3-6	(2,4,6,8)	(1500,2000,2500,3000)	(1,3,5,7)	(4,3,5,7)
5-7	(5,8,11,14)	(700,1200,1700,2200)	(1,3,5,7)	(2,3,4,5)
4-8	(4,5,6,7)	(1000,1500,2000,2500)	(5,7,9,11)	(3,4,5,6)
6-10	(3,6,9,12)	(900,2000,2500,5000)	(3,5,7,9)	(2,3,4,5)
7-10	(3,5,7,9)	(3500,4000,4500,5000)	(5,7,9,11)	(3,4,5,6)
9-10	(4,5,6,7)	(2500,3000,3500,4000)	(5,7,9,11)	(1,2,3,4)
Weight	**(7,9,9,9)**	**(7,9,9,9)**	**(5,7,9,11)**	**(5,7,9,11)**

Table 2 Criterias Time, Cost, Risk and Quality Converted in to trapezoidal fuzzy numbers

Activity	Time	Cost	Risk	Quality
1-2	(28,63,90,108)	(10500,18000,22500,27000)	(25,49,81,121)	(15,14,18,33)
1-3	(21,54,81,108)	(24500,9000,13500,18000)	(5,21,45,77)	(5,0,0,11)
1-4	(14,36,54,72)	(1400,6300,10800,15300)	(5,21,45,77)	(10,7,9,22)
2-5	(21,45,63,81)	(1400,6300,10800,15300)	(15,35,63,99)	(5,0,0,11)
3-5	(21,36,45,54)	(10500,18000,22500,27000)	(15,35,63,99)	(5,0,0,11)
4-6	(14,27,36,45)	(10500,18000,22500,27000)	(15,35,63,99)	(10,7,9,22)
8-9	(56,90,108,126)	(38500,54000,58500,63000)	(5,21,45,77)	(10,7,9,22)
3-6	(14,36,54,72)	(10500,18000,22500,27000)	(5,21,45,77)	(0,7,0,0)
5-7	(35,72,99,126)	(4900,10800,15300,19800)	(5,21,45,77)	(10,7,9,22)
4-8	(28,45,54,63)	(7000,13500,18000,22500)	(25,49,81,121)	(5,0,0,11)
6-10	(21,54,81,108)	(6300,18000,22500,45000)	(15,35,63,99)	(10,7,9,22)
7-10	(21,45,63,81)	(24500,36000,40500,45000)	(25,49,81,121)	(5,0,0,11)
9-10	(28,45,54,63)	(17500,27000,31500,36000)	(25,49,81,121)	(15,14,18,33)

Table 3 Time, Cost, Risk and Quality multiplied by corresponding weights

Activity	**Time**	**Cost**	**Risk**	**Quality**
1-2	(30.96,63,90,106.2)	(11160,18000,22500,26550)	(27.04,49,81,116.64)	(14.56,14,18,31.32)
1-3	(23.76,54,81,105.3)	(23400,9000,13500,17550)	(6.24,21,45,73.44)	(4.16,0,0,9.72)
1-4	(15.84,36,54,70.2)	(1800,6300,10800,14850)	(6.24,21,45,73.44)	(9.36,7,9,20.52)
2-5	(23.04,45,63,79.2)	(1800,6300,10800,14850)	(16.64,35,63,95.04)	(4.16,0,0,9.72)
3-5	(22.32,36,45,53.1)	(11160,18000,22500,26550)	(16.64,35,63,95.04)	(4.16,0,0,9.72)
4-6	(15.12,27,36,44.1)	(39960,54000,58500,62550)	(16.64,35,63,95.04)	(9.36,7,9,20.52)
8-9	(59.04,90,108,124.2)	(11160,18000,22500,26550)	(6.24,21,45,73.44)	(9.36,7,9,20.52)
3-6	(15.84,36,54,70.2)	(11160,18000,22500,26550)	(6.24,21,45,73.44)	(0,7,0,0)
5-7	(38.16,72,99,123.3)	(5400,10800,15300,19350)	(6.24,21,45,73.44)	(9.36,7,9,20.52)
4-8	(29.52,45,54,62.1)	(7560,13500,18000,22050)	(27.04,49,81,116.64)	(4.16,0,0,9.72)
6-10	(23.76,54,81,105.3)	(7272,18000,22500,42750)	(16.64,35,63,95.04)	(9.36,7,9,20.52)
7-10	(23.04,45,63,79.2)	(25560,36000,40500,44550)	(27.04,49,81,116.64)	(4.16,0,0,9.72)
9-10	(29.52,45,54,62.1)	(18360,27000,31500,35550)	(27.04,49,81,116.64)	(14.56,14,18,31.32)

Table 4 Time, Cost, Risk and Quality with $\alpha = 0.1$ multiplied by corresponding weights

Activity	Time	Cost	Risk	Quality
1-2	(44,63,90,99)	(14000,18000,22500,24750)	(36,49,81,100)	(12,14,18,25)
1-3	(36,54,81,94.5)	(18000,9000,13500,15750)	(12,21,45,60)	(0,0,0,5)
1-4	(24,36,54,63)	(3600,6300,10800,13050)	(12,21,45,60)	(6,7,9,15)
2-5	(32,45,63,72)	(3600,6300,10800,13050)	(24,35,63,80)	(0,0,0,5)
3-5	(28,36,45,49.5)	(14000,18000,22500,24750)	(24,35,63,80)	(0,0,0,5)
4-6	(20,27,36,40.5)	(46000,54000,58500,60750)	(24,35,63,80)	(6,7,9,15)
8-9	(72,90,108,117)	(14000,18000,22500,24750)	(12,21,45,60)	(6,7,9,15)
3-6	(24,36,54,63)	(14000,18000,22500,24750)	(12,21,45,60)	(0,7,0,0)
5-7	(52,72,99,112.5)	(7600,10800,15300,17550)	(12,21,45,60)	(6,7,9,15)
4-8	(36,45,54,58.5)	(10000,13500,18000,20250)	(36,49,81,100)	(0,0,0,5)
6-10	(36,54,81,94.5)	(11600,18000,22500,33750)	(24,35,63,80)	(6,7,9,15)
7-10	(32,45,63,72)	(30000,36000,40500,42750)	(36,49,81,100)	(0,0,0,5)
9-10	(36,45,54,58.5)	(22000,27000,31500,33750)	(36,49,81,100)	(12,14,18,25)

Table 5 Time, Cost, Risk and Quality with $\alpha = 0.5$ multiplied by corresponding weights

Activity	Time	Cost	Risk	Quality
1-2	(58.96,63,90,91.8)	(17160,18000,22500,22950)	(46.24,49,81,84.64)	(13.6,14,18,19.32)
1-3	(50.16,54,81,83.7)	(11000,9000,13500,13950)	(19.04,21,45,47.84)	(0,0,0,0.92)
1-4	(33.44,36,54,55.8)	(5720,6300,10800,11250)	(19.04,21,45,47.84)	(6.8,7,9,10.12)
2-5	(42.24,45,63,64.8)	(5720,6300,10800,11250)	(32.64,35,63,66.24)	(0,0,0,0.92)
3-5	(34.32,36,45,45.9)	(17160,18000,22500,22950)	(32.64,35,63,66.24)	(0,0,0,0.92)
4-6	(25.52,27,36,36.9)	(52360,54000,58500,58950)	(32.64,35,63,66.24)	(6.8,7,9,10.12)
8-9	(86.24,90,108,109.8)	(17160,18000,22500,22950)	(19.04,21,45,47.84)	(6.8,7,9,10.12)
3-6	(33.44,36,54,55.8)	(17160,18000,22500,22950)	(19.04,21,45,47.84)	(5.44,7,0,0)
5-7	(67.76,72,99,101.7)	(10120,10800,15300,15750)	(19.04,21,45,47.84)	(6.8,7,9,10.12)
4-8	(43.12,45,54,54.9)	(12760,13500,18000,18450)	(46.24,49,81,84.64)	(0,0,0,0.92)
6-10	(50.16,54,81,83.7)	(16632,18000,22500,24750)	(32.64,35,63,66.24)	(6.8,7,9,10.12)
7-10	(42.24,45,63,64.8)	(34760,36000,40500,40950)	(46.24,49,81,84.64)	(0,0,0,0.92)
9-10	(43.12,45,54,54.9)	(25960,27000,31500,31950)	(46.24,49,81,84.64)	(13.6,14,18,19.32)

Table 6 Time, Cost, Risk and Quality with $\alpha = 0.9$ multiplied by corresponding weights

12.1 The mathematical formulation of FMOCPP is as follows:

Formulation of Model 2

$$\begin{aligned} \textit{Minimize } z_1(Time) = {} & (28,63,90,108)\times x_{12} + (21,54,81,108)\times x_{13} + (14,36,54,72)\times x_{14} + \\ & (21,45,63,81)\times x_{25} + (21,36,45,54)\times x_{35} + (14,27,36,45)\times x_{46} + \\ & (56,90,108,126)\times x_{89} + (14,36,54,72)\times x_{36} + (35,72,99,126)\times x_{57} + \\ & (28,45,54,63)\times x_{48} + (21,54,81,108)\times x_{610} + (21,45,63,81)\times x_{710} + \\ & (28,45,54,63)\times x_{910}, \end{aligned}$$

$$\begin{aligned} \text{Minimize } z_2(Cost) = {} & (10500,18000,22500,27000)\times x_{12} + (7000,13500,18000,31500)\times x_{13} + \\ & (1400,6300,10800,15300)\times x_{14} + (1400,6300,10800,15300)\times x_{25} + \\ & (10500,18000,22500,27000)\times x_{35} + (10500,18000,22500,27000)\times x_{46} + \\ & (38500,54000,58500,63000)\times x_{89} + (10500,18000,22500,27000)\times x_{36} + \\ & (4900,10800,15300,19800)\times x_{57} + (7000,13500,18000,22500)\times x_{48} + \\ & (6300,18000,22500,45000)\times x_{610} + (24500,36000,40500,45000)\times x_{710} + \\ & (17500,27000,31500,36000)\times x_{910}, \end{aligned}$$

$$\begin{aligned} \textit{Minimize } z_3(Quality) = {} & (15,14,18,33)\times x_{12} + (5,0,0,11)\times x_{13} + (10,7,9,22)\times x_{14} + (5,0,0,11)\times x_{25} + \\ & (5,0,0,11)\times x_{35} + (10,7,9,22)\times x_{46} + (10,7,9,22)\times x_{89} + (0,7,0,0)\times x_{36} + \\ & (10,7,9,22)\times x_{57} + (5,0,0,11)\times x_{48} + (10,7,9,22)\times x_{610} + (5,0,0,11)\times x_{710} + \\ & (15,14,18,33)\times x_{910}, \end{aligned}$$

$$\begin{aligned} \textit{Minimize } z_4(Risk) = {} & (25,49,81,121)\times x_{12} + (5,21,45,77)\times x_{13} + (5,21,45,77)\times x_{14} + \\ & (15,35,63,99)\times x_{25} + (15,35,63,99)\times x_{35} + (15,35,63,99)\times x_{46} + \\ & (5,21,45,77)\times x_{89} + (5,21,45,77)\times x_{36} + (5,21,45,77)\times x_{57} + \\ & (25,49,81,121)\times x_{48} + (15,35,63,99)\times x_{610} + (25,49,81,121)\times x_{710} + \\ & (25,49,81,121)\times x_{910}. \end{aligned}$$

Subject to the constraints ,

$$x_{12}+x_{13}+x_{14}=1$$
$$x_{12}=x_{25}$$
$$x_{13}=x_{35}+x_{36}$$
$$x_{14}=x_{46}+x_{48}$$
$$x_{25}+x_{35}=x_{57}$$
$$x_{36}+x_{46}=x_{610}$$
$$x_{57}=x_{710}$$
$$x_{48}=x_{89}$$
$$x_{89}=x_{910}$$
$$x_{610}+x_{710}+x_{910}=1,$$
$$x_{12}\geq 0,\ x_{13}\geq 0,\ x_{14}\geq 0, x_{25}\geq 0, x_{35}\geq 0, x_{36}\geq 0, x_{46}\geq 0, x_{48}\geq 0, x_{57}\geq 0, x_{89}$$
$$\geq 0, x_{610}\geq 0, x_{710}\geq 0, x_{910}\geq 0.$$

12.2 Solution

For finding the solution of this fuzzy project network analysis problem the fuzzy programming technique based developed approach is utilized and for that at different α level the value of each objective PIS and NIS are as Table 7.

α level	Solut-ions	Objective												
		z_{11}	z_{12}	z_{13}	z_{14}	z_{21}	z_{22}	z_{23}	z_{24}	z_{31}	z_{32}	z_{33}	z_{34}	z_{41}
0	PIS	49	117	171	225	18200	42300	55800	87300	15	7	9	33	25
	NIS	126	225	315	396	64400	100800	118800	136800	40	28	36	88	70
0.1	PIS	54.72	117	171	219.6	25992	42300	55800	99000	13.52	7	9	30.24	29.12
	NIS	133.92	225	315	387.9	49680	100800	118800	120600	37.44	28	36	82.08	76.96
0.5	PIS	80	117	171	198	35600	42300	55800	83250	6	7	9	20	48

	NIS	168	225	315	355.5	61600	100800	118800	109800	24	28	36	60	108
0.9	PIS	109.12	117	171	176.4	46552	42300	55800	67050	6.8	7	9	11.04	70.72
	NIS	211.2	225	315	323.1	74800	100800	118800	99000	27.2	28	36	40.48	144.16

Table 7 Positive ideal solution (PIS) and Negative ideal solution (NIS) for each objective

Substituting the values acquired in Table 7 in Model 3.2, we get

Maximize λ,

subject to the constraints:

$77\times\lambda+28\times x_{12}+21\times x_{13}+14\times x_{14}+21\times x_{25}+21\times x_{35}+14\times x_{46}+56\times x_{89}+14\times x_{36}+$
$35\times x_{57}+28\times x_{48}+21\times x_{610}+21\times x_{710}+28\times x_{910}\leq 126,$

$46200\times\lambda+10500\times x_{12}+7000\times x_{13}+1400\times x_{14}+1400\times x_{25}+10500\times x_{35}+10500\times x_{46}+$
$38500\times x_{89}+10500\times x_{36}+4900\times x_{57}+7000\times x_{48}+6300\times x_{610}+24500\times x_{710}+17500\times x_{910}$
$\leq 64400,$

$45\times\lambda+25\times x_{12}+5\times x_{13}+5\times x_{14}+15\times x_{25}+15\times x_{35}+15\times x_{46}+5\times x_{89}+5\times x_{36}+5\times x_{57}+$
$25\times x_{48}+15\times x_{610}+25\times x_{710}+25\times x_{910}\leq 70,$

$25\times\lambda+15\times x_{12}+5\times x_{13}+10\times x_{14}+5\times x_{25}+5\times x_{35}+10\times x_{46}+10\times x_{89}+0\times x_{36}+10\times x_{57}+$
$5\times x_{48}+10\times x_{610}+5\times x_{710}+15\times x_{910}\leq 40,$

$108\times\lambda+63\times x_{12}+54\times x_{13}+36\times x_{14}+45\times x_{25}+36\times x_{35}+27\times x_{46}+90\times x_{89}+36\times x_{36}+$
$72\times x_{57}+45\times x_{48}+54\times x_{610}+45\times x_{710}+45\times x_{910}\leq 225,$

$58500\times\lambda+18000\times x_{12}+13500\times x_{13}+6300\times x_{14}+6300\times x_{25}+18000\times x_{35}+18000\times x_{46}$
$+54000\times x_{89}+18000\times x_{36}+10800\times x_{57}+13500\times x_{48}+18000\times x_{610}+36000\times x_{710}+$
$27000\times x_{910}\leq 100800,$

$77\times\lambda+49\times x_{12}+21\times x_{13}+21\times x_{14}+35\times x_{25}+35\times x_{35}+35\times x_{46}+21\times x_{89}+21\times x_{36}+$
$21\times x_{57}+49\times x_{48}+35\times x_{610}+49\times x_{710}+49\times x_{910}\leq 154,$

$21\times\lambda+14\times x_{12}+0\times x_{13}+7\times x_{14}+0\times x_{25}+0\times x_{35}+7\times x_{46}+7\times x_{89}+7\times x_{36}+7\times x_{57}+$
$0\times x_{48}+7\times x_{610}+0\times x_{710}+14\times x_{910}\leq 28,$

$144\times\lambda+90\times x_{12}+81\times x_{13}+54\times x_{14}+63\times x_{25}+45\times x_{35}+36\times x_{46}+108\times x_{89}+54\times x_{36}+99\times x_{57}+54\times x_{48}+81\times x_{610}+63\times x_{710}+54\times x_{910}\leq 315,$

$63000\times\lambda+22500\times x_{12}+18000\times x_{13}+10800\times x_{14}+10800\times x_{25}+22500\times x_{35}+22500\times x_{46}+58500\times x_{89}+22500\times x_{36}+15300\times x_{57}+18000\times x_{48}+22500\times x_{610}+40500\times x_{710}+31500\times x_{910}\leq 118800,$

$117\times\lambda+81\times x_{12}+45\times x_{13}+45\times x_{14}+63\times x_{25}+63\times x_{35}+63\times x_{46}+45\times x_{89}+45\times x_{36}+45\times x_{57}+81\times x_{48}+63\times x_{610}+81\times x_{710}+81\times x_{910}\leq 270,$

$27\times\lambda+18\times x_{12}+0\times x_{13}+9\times x_{14}+0\times x_{25}+0\times x_{35}+9\times x_{46}+9\times x_{89}+0\times x_{36}+9\times x_{57}+0\times x_{48}+9\times x_{610}+0\times x_{710}+18\times x_{910}\leq 36,$

$171\times\lambda+108\times x_{12}+108\times x_{13}+72\times x_{14}+81\times x_{25}+54\times x_{35}+45\times x_{46}+126\times x_{89}+72\times x_{36}+126\times x_{57}+63\times x_{48}+108\times x_{610}+81\times x_{710}+63\times x_{910}\leq 396,$

$49500\times\lambda+27000\times x_{12}+31500\times x_{13}+15300\times x_{14}+15300\times x_{25}+27000\times x_{35}+27000\times x_{46}+63000\times x_{89}+27000\times x_{36}+19800\times x_{57}+22500\times x_{48}+45000\times x_{610}+45000\times x_{710}+36000\times x_{910}\leq 136800,$

$165\times\lambda+121\times x_{12}+77\times x_{13}+77\times x_{14}+99\times x_{25}+99\times x_{35}+99\times x_{46}+77\times x_{89}+77\times x_{36}+77\times x_{57}+121\times x_{48}+99\times x_{610}+121\times x_{710}+121\times x_{910}\leq 418,$

$55\times\lambda+33\times x_{12}+11\times x_{13}+22\times x_{14}+11\times x_{25}+11\times x_{35}+22\times x_{46}+22\times x_{89}+0\times x_{36}+22\times x_{57}+11\times x_{48}+22\times x_{610}+11\times x_{710}+33\times x_{910}\leq 88,$

with constraints of this problem.

The solution of this model by developed approach using LINGO software is given in Table 8.

α level	λ	Optimal path	(z_1, z_2, z_3, z_4)	Optimal path in [35] at $\alpha=0$	Objective values (z_1, z_2, z_3, z_4) in [35] at $\alpha=0$
$\alpha=0$	0.6316	1-3-6-10	((8,16,24,32), (3400,5500,7000,11500), (5,11,17,23),(9,10,14,18))	1-3-6-10	((8,16,24,32), (3400,5500,7000,11500), (5,11,17,23),(9,10,14,18))
$\alpha=0.1$	0.6364	1-3-6-10	((8.8,16,24,31.2), (3610,5500,7000,11050), (5.6,11,17,22.4),(9.1,10,14,17.6))		

$\alpha = 0.5$	0.6571	1-3-6-10	((12,16,24,28), (4450,5500,7000,9250), (8,11,17,20),(9.5,10,14,16))		
$\alpha = 0.9$	0.6667	1-3-6-10	((15.2,16,24,24.8), (5290,5500,7000,7450), (10.4,11,17,17.6),(9.9,10,14,14.4))		

Table 8 Results for $\alpha = 0, \alpha = 0.1, \alpha = 0.5$ *and* $\alpha = 0.9$ **using model 3.2**

Table-8 indicate the solution of illustrated FMOCPP, which shows that at α level 0, 0.1, 0.5 and 0.9 the optimal degree of satisfaction are 0.6316, 0.6364, 0.6571 and 0.6667respectively. Table-7 also indicate that critical path remain same at each α level. Table8 also compares the developed solution approach with other existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Figure 4indicate shows the distribution of objective values with respect to liner membership function at different α level.

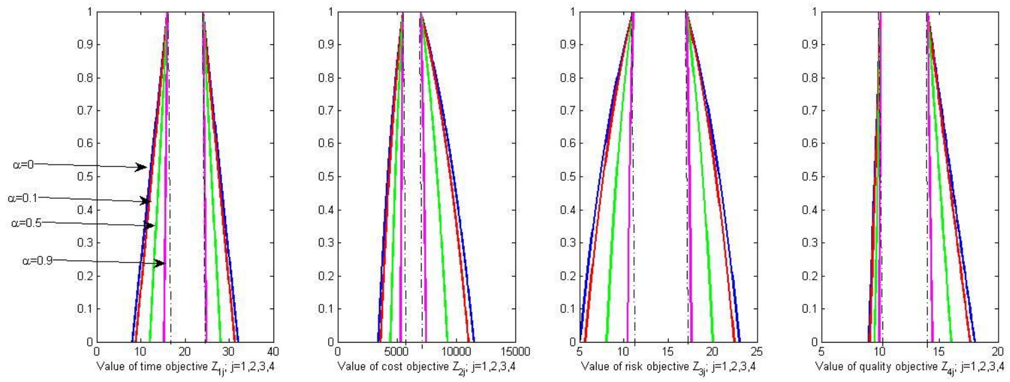

Figure 4 Time,cost,risk and quality objective at α **levels 0, 0.1, 0.5 and 0.9 with linear membership model 3.2**

Solution method using Model 4 (Exponential Membership model)

Model 4 can be formulated with PIS and NIS obtained in Table 7 as follow:

Maximize λ,

subject to the constraints:

$$\exp(-s((28\times x_{12}+21\times x_{13}+14\times x_{14}+21\times x_{25}+21\times x_{35}+14\times x_{46}+56\times x_{89}+14\times x_{36}+35\times x_{57}+28\times x_{48}+21\times x_{610}+21\times x_{710}+28\times x_{910})-49)/77)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((10500\times x_{12}+7000\times x_{13}+1400\times x_{14}+1400\times x_{25}+10500\times x_{35}+10500\times x_{46}+38500\times x_{89}+10500\times x_{36}+4900\times x_{57}+7000\times x_{48}+6300\times x_{610}+24500\times x_{710}+17500\times x_{910})-18200)/46200)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((25\times x_{12}+5\times x_{13}+5\times x_{14}+15\times x_{25}+15\times x_{35}+15\times x_{46}+5\times x_{89}+5\times x_{36}+5\times x_{57}+25\times x_{48}+15\times x_{610}+25\times x_{710}+25\times x_{910})-25)/45)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((15\times x_{12}+5\times x_{13}+10\times x_{14}+5\times x_{25}+5\times x_{35}+10\times x_{46}+10\times x_{89}+0\times x_{36}+10\times x_{57}+5\times x_{48}+10\times x_{610}+5\times x_{710}+15\times x_{910})-15)/25)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((63\times x_{12}+54\times x_{13}+36\times x_{14}+45\times x_{25}+36\times x_{35}+27\times x_{46}+90\times x_{89}+36\times x_{36}+72\times x_{57}+45\times x_{48}+54\times x_{610}+45\times x_{710}+45\times x_{910})-117)/108)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((18000\times x_{12}+13500\times x_{13}+6300\times x_{14}+6300\times x_{25}+18000\times x_{35}+18000\times x_{46}+54000\times x_{89}+18000\times x_{36}+10800\times x_{57}+13500\times x_{48}+18000\times x_{610}+36000\times x_{710}+27000\times x_{910})-42300)/58500)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((49\times x_{12}+21\times x_{13}+21\times x_{14}+35\times x_{25}+35\times x_{35}+35\times x_{46}+21\times x_{89}+21\times x_{36}+21\times x_{57}+49\times x_{48}+35\times x_{610}+49\times x_{710}+49\times x_{910})-77)/77)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((14\times x_{12}+0\times x_{13}+7\times x_{14}+0\times x_{25}+0\times x_{35}+7\times x_{46}+7\times x_{89}+7\times x_{36}+7\times x_{57}+0\times x_{48}+7\times x_{610}+0\times x_{710}+14\times x_{910})-7)/21)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((90\times x_{12}+81\times x_{13}+54\times x_{14}+63\times x_{25}+45\times x_{35}+36\times x_{46}+108\times x_{89}+54\times x_{36}+99\times x_{57}+54\times x_{48}+81\times x_{610}+63\times x_{710}+54\times x_{910})-171)/144)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((22500\times x_{12}+18000\times x_{13}+10800\times x_{14}+10800\times x_{25}+22500\times x_{35}+22500\times x_{46}+58500\times x_{89}+22500\times x_{36}+15300\times x_{57}+18000\times x_{48}+22500\times x_{610}+40500\times x_{710}+31500\times x_{910})-55800)/63000)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((81\times x_{12}+45\times x_{13}+45\times x_{14}+63\times x_{25}+63\times x_{35}+63\times x_{46}+45\times x_{89}+45\times x_{36}+45\times x_{57}+81\times x_{48}+63\times x_{610}+81\times x_{710}+81\times x_{910})-153)/117)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((18\times x_{12}+0\times x_{13}+9\times x_{14}+0\times x_{25}+0\times x_{35}+9\times x_{46}+9\times x_{89}+0\times x_{36}+9\times x_{57}+0\times x_{48}+9\times x_{610}+0\times x_{710}+18\times x_{910})-9)/27)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((108\times x_{12}+108\times x_{13}+72\times x_{14}+81\times x_{25}+54\times x_{35}+45\times x_{46}+126\times x_{89}+72\times x_{36}+126\times x_{57}+63\times x_{48}+108\times x_{610}+81\times x_{710}+63\times x_{910})-225)/171)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$$\exp(-s((27000\times x_{12}+31500\times x_{13}+15300\times x_{14}+15300\times x_{25}+27000\times x_{35}+27000\times x_{46}+63000\times x_{89}+27000\times x_{36}+19800\times x_{57}+22500\times x_{48}+45000\times x_{610}+45000\times x_{710}+36000\times x_{910})-87300)/49500)-((1-\exp(-s))\times\lambda)\geq\exp(-s),$$

$\exp(-s((121\times x_{12}+77\times x_{13}+77\times x_{14}+99\times x_{25}+99\times x_{35}+99\times x_{46}+77\times x_{89}+77\times x_{36}+$
$77\times x_{57}+121\times x_{48}+99\times x_{610}+121\times x_{710}+121\times x_{910})-253)/165)-((1-\exp(-s))\times\lambda)$
$\geq \exp(-s),$
$\exp(-s((33\times x_{12}+11\times x_{13}+22\times x_{14}+11\times x_{25}+11\times x_{35}+22\times x_{46}+22\times x_{89}+0\times x_{36}+$
$22\times x_{57}+11\times x_{48}+22\times x_{610}+11\times x_{710}+33\times x_{910})-33)/55)-((1-\exp(-s))\times\lambda)\geq$
$\exp(-s),$

subject to the constraints of this problem.

We have considered different values of shape parameters as in table 9.

Case	Shape parameter (s_1, s_2, s_3, s_4)
Case -1	(-1,-1,-1,-1)
Case -2	(-0.1,-0.3,-0.6,-0.8)
Case -3	(-0.1,-0.4,-0.8,-0.9)
Case -4	(-0.2,-0.4,-0.7,-0.9)
Case -5	(-0.1,-0.3,-0.6,-1)

Table 9 Shape parameters

The solution of this exponential models with different values of shape parameters of table 9 by using LINGO software is given in Table 10.

α level	Case	Degree of satisfaction λ	Optimal path	(z_1, z_2, z_3, z_4)	Optimal path in research paper[35]	Objective values (z_1, z_2, z_3, z_4) at $\alpha=0$ in research paper [35]
$\alpha=0$	Case -1	0.7406	1-3-6-10	((8,16,24,32), (3400,5500,7000,11500),		
	Case -2	0.6430	1-3-6-10			

	Case - 3	0.6430	1-3-6-10	(5,11,17,23), (9,10,14,18))	1-3-6-10	((8,16,24,32), (3400,5500,7000,11500), (5,11,17,23), (9,10,14,18))
	Case - 4	0.6545	1-3-6-10			
	Case - 5	0.6430	1-3-6-10			
$\alpha = 0.1$	Case - 1	0.7449	1-3-6-10	((8.8,16,24,31.2), (3610,5500,7000,11050), (5.6,11,17,22.4), (9.1,10,14,17.6))		
	Case - 2	0.6480	1-3-6-10			
	Case - 3	0.6480	1-3-6-10			
	Case - 4	0.6594	1-3-6-10			
	Case - 5	0.6480	1-3-6-10			
$\alpha = 0.5$	Case - 1	0.7620	1-3-6-10	((12,16,24,28), (4450,5500,7000,9250), (8,11,17,20), (9.5,10,14,16))		
	Case - 2	0.6683	1-3-6-10			
	Case - 3	0.6683	1-3-6-10			
	Case - 4	0.6794	1-3-6-10			
	Case - 5	0.6683	1-3-6-10			
$\alpha = 0.9$	Case - 1	0.7694	1-3-6-10	((15.2,16,24,24.8), (5290,5500,7000,7450), (10.4,11,17,17.6), (9.9,10,14,14.4))		
	Case - 2	0.6916	1-3-6-10			
	Case - 3	0.6916	1-3-6-10			
	Case -	0.7023	1-3-6-10			

	4					
	Case - 5	0.6916	1-3-6-10			

Table 10 Results for $\alpha = 0, \alpha = 0.1, \alpha = 0.5 \, and \, \alpha = 0.9$ **using model 4**

Table 10 indicate the solution of FMOCPP with exponential membership function by fuzzy programming technique. It shows that at α level 0 the optimal degree of satisfaction are 0.7406, 0.6430, 0.6430, 0.6545 and 0.6430 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. Similarly at α level 0.1 the optimal degree of satisfaction are 0.7449,0.6480,0.6480,0.6594 and 0.6480 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. For α level 0.5 the optimal degree of satisfaction are 0.7620,0.6683,0.6683,0.6794 and 0.6683 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. Similarly For α level 0.9 the optimal degree of satisfaction are 0.7694,0.6916,0.6916,0.7023 and 0.6916 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. With five cases of shape parameter we will get optimal path with different degree of satisfaction which provides opportunity to DM to take the decisions. If decision makers are not satisfied with obtain critical path they may change the different value of shape parameters to obtain desired level of satisfaction and this is one of the best advantage of this developed approach. Table-10 also compares the obtained output with existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The figure 5 indicates the distribution of objective values with respect to liner membership function at different α level.

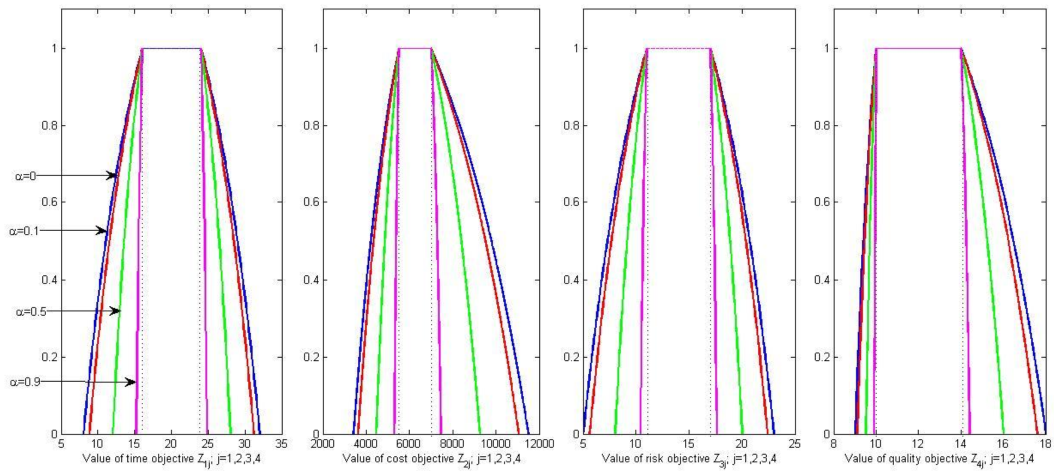

Figure 5 Time,cost,risk and quality objective at α levels 0, 0.1, 0.5 and 0.9 with exponentaial membership model 4

13 Conclusion

This approach provides the solution of FMOCPP using fuzzy linear membership function and exponential membership function subject to some realistic constraints to optimize the optimistic, the most likely and the pessimistic scenario of fuzzy objective functions with the trapezoidal possibilistic distribution. The main benefit of our new approach is it provides optimum critical path according to all criteria's without calculation of all performance ranking of each path also sensitivity analysis is not necessary to perform in this approach.

REFERENCES

[1] Amiri, M. Golozari, F,"Application of fuzzy multi-attribute decision making in determining the critical path by using time,cost, risk, and quality criteria", Int J Adv Manuf Technol, 54, pp.393– 401, 2011.

[2] Ashtiani, B., Haghighirad, F., Makui, A. and Montazer, G. A., "Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets", Applied Soft Computing, Vol. 9, No. 2, pp. 457-461, 2008.

[3] Azaron, A., Katagiri, H., Sakawa, M., Kato, K. and Memariani, A., “A multi-objective resource allocationproblem in PERT networks”, European Journal of Operational Research, 172, pp. 838–854, 2006.

[4] Benitez, J. M., Martin, J. C. and Roman, C., “Using fuzzy number for measuring quality of service in the hotel industry”, Tourism Management, Vol. 28, No. 2, pp. 544-555, 2007.

[5] Chanas, S., Dubois, D. and Zielin´ski, P., “On the sure criticality of tasks in activity networks with imprecise durations”, IEEE Transactions on Systems, Man, and Cybernetics – Part B, Cybernetics 32, pp. 393–407, 2002.

[6] Chanas, S. and Kamburowski, J., “The use of fuzzy variables in PERT”, Fuzzy Sets and Systems, Vol.5, pp. 11–19, 1981.

[7] Chanas, S. and Zielinski, P. “Critical path analysis in the network with fuzzy task times”, Fuzzy Sets and Systems, 122, pp.195–204, 2001.

[8] Chanas, S. and Zielin´ski, P., “ The computational complexity of the criticality problems in a network with interval activity times”, European Journal of Operational Research 136, pp. 541–550, 2002.

[9] Chanas, S. and P. Zielinski, “On the hardness of evaluating criticality of activities in a planar network with duration intervals”, Operations Research Letters, 31, pp. 53–59, 2003.

[10] Chen CT, Huang SF, “Applying fuzzy method for measuring criticality in project network”, Inf Sci., 177, pp.2448– 2458, 2007.

[11] Chen, C. T., Lin, C. T. and Huang, S. F., “A fuzzy approach for supplier evaluation and selection in supply chain management”, International Journal of Production Economics, Vol. 102, No. 2, pp. 289-301, 2006.

[12] Chen, S.M. and Chang, T.H., “Finding multiple possible critical paths using fuzzy PERT”, IEEE Transactions on Systems, man and Cybernetics – Part B, Cybernetics 31, pp. 930–937, 2001.

[13] Chen, SP. “Analysis of critical paths in a project network with fuzzy activity times”,Eur J Oper Res 183, pp.442–459, 2007

[14] Chen, SP. and Hsueh, YJ., “A simple approach to fuzzy critical path analysis in project networks’, Appl Math Model, 32 (7), pp. 1289–1297,2008.

[15] Chen,T.C.,“Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems”, Volume 114, Issue 1, 16, pp.1-9, 2000.

[16] Chen, T.Y. and Tsao, C.Y., "The interval-valued fuzzy TOPSIS method and experimental analysis, Fuzzy Sets and Systems", Vol. 159, No. 11, pp. 1410-1428, 2008.

[17] Chu, T. C. and Lin, Y. C., "An interval arithmetic based fuzzy TOPSIS model, Expert Systems with Applications", Vol. 36, No. 8, pp. 10870-10876, 2009.

[18] Cristobal, JRS., "Critical path definition using multicriteria decision making: PROMETHEE method", J Manag Eng 29, pp. 158–163, 2013.

[19] Dubois, D., Fargier,H. and Galvagnon, V. "On latest starting times and floats in activity networks with ill- known durations", European Journal of Operational Research, 147, pp.266–280, 2003.

[20] Elizabeth, S. and Sujatha, L., "Fuzzy critical path problem for project network", Int J Pure Appl Math, 85, pp.223– 240, 2013.

[21] Ekmekcioglu, M., Kaya, T. and Kahraman, C.,"Fuzzy multi-criteria disposal method and site selection for municipal solid waste",Waste Management, Vol. 30, No. 8-9, pp. 1729-1736, 2010.

[22] Hassanzadeh, R., Amiri, N.M., Tajdin, A. "An α -Cut Approach for Fuzzy Product and Its Use in Computing Solutions of Fully Fuzzy Linear Systems", Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management, Istanbul, Turkey, July 3-6, 2012 Jahanshahloo, G. R. Hosseinzadeh L, F. and Izadikhah, M., "Extension of the TOPSIS method for decision-making problems with fuzzy data", Applied Mathematics and Computation, Vol. 181, No. 2, pp. 1544–1551, 2006.

[23] Jahanshahloo, G. R. Hosseinzadeh L, F. and Izadikhah, M., "Extension of the TOPSIS method for decision- making problems with fuzzy data", Applied Mathematics and Computation, Vol. 181, No. 2, pp. 1544–1551, 2006.

[24] Kahraman, C., Cevik, S., Ates, N. Y. and Gulbay, M., "Fuzzy multi-criteria evaluation of industrial robotic systems", Computers & Industrial Engineering, Vol. 52, No. 4, pp. 414-433, 2007.

[25] Kannan G., Shaligram P., Kumar, P. S., "A hybrid approach using ISM and fuzzy TOPSIS for the selection of reverse logistics provider", Resour. Conserv. Recycl, 54, pp. 28–36, 2009.

[26] Kaya, T. and Kahraman, C., "Multi-criteria decision making in energy planning using a modified fuzzy TOPSIS methodology", Expert Systems with Applications, Vol. 38, No. 6, pp. 6577-6585, 2011.

[27] Kelly J.,"Critical-path planning and scheduling: Mathematical basis",Operations Research, 9(3), pp.296–320, 1961.

[28] Kim, S., Lee, K., Cho, J. K. and Kim, C.O., "Agent-based diffusion model for an automobile market with fuzzy TOPSIS-based product adoption process", Expert Systems with Applications, Vol. 38, No. 6, pp. 7270-7276, 2011.

[29] Kutlu, A. C. and Ekmekcioglu, M., "Fuzzy failure modes and effects analysis by using fuzzy TOPSIS integrated with fuzzy AHP", Expert Systems with Applications, Vol. 39, No. 1, pp. 61-67, 2011.

[30] Liang, G. S., "Fuzzy MCDM based on ideal and anti-ideal concepts", European Journal of Operational Research, Vol. 112, No. 3, pp. 682-691, 1999.

[31] Lin, CT. and Ying, TC. "Bid no bid decision making, a fuzzy linguistic approach". Int J Proj Manag 22, pp.585– 593, 2004.

[32] Mahdi, I. and Alreshaid, K., "Decision support system for select the proper project delivery method using analytical hierarchy process (AHP)", Int J Proj Manag., 23, pp.564–572, 2005.

[33] Mehlawat, MK and Gupta, P., "A new fuzzy group multi-criteria decision making method with an application to the critical path selection", Int J AdvManufTechnol, 83, pp. 1281–1296, 2016.

[34] Rommelfanger, H., " Interactive decision making in fuzzy linear optimization problems", European Journal of Operational Research, 41(2), pp. 210–217, 1989.

[35] Saradhi, B. P., Shankar, N. R. and Suryanarayana, CH. " Selection of critical path in a project network using topsis method", International Journal of Mathematical Archive, 6(10), pp.187-196, 2015.

[36] Sharma, J.K., "Operation Research Theory and Applications", 3rd Edition, Macmillan, pp.483-484, 2007.

[37] Tailor, A.R and Dhodiya, J.M , "Genetic algorithm based hybrid approach to solve optimistic, most-likely and pessimistic scenarios of fuzzy multi-objective assignment problem using exponential membership function, British Journal of Mathematics and Computer Science, 17(2), pp. 1–19, 2016a.

[38] Tanaka, H., Ichihashi, H. and Asai, K., " A formulation of fuzzy linear programming problem based on comparison of fuzzy numbers", Control and Cybernetics, 13, pp. 185–194,1984.

[39] Wang, T. C. and Chang, T. H., "Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment", Expert Systems with Applications, Vol. 33, No. 4, pp. 870-880, 2007.

[40] Yong, D. , "Plant location selection based on fuzzy TOPSIS", International Journal of Advanced Manufacturing Technologies, Vol. 28, No. 7-8, pp. 323-326, 2006.

[41] Yuan Y., "Criteria for evaluating fuzzy ranking methods", Fuzzy Sets Syst. 43, pp.139–157, 1991.

[42] Zadeh LA, Fuzzy sets. Inf Control 8(3), pp.338–353, 1965.