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UNIDOMINATING FUNCTIONS OF ROOTED PRODUCT OF $P_m o P_n$

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ABSTRACT:

Unidominating function concept was introduced by V. Anantha Lakshmi and B. Maheshwari in 2015. In this paper we present unidominating functions for root product of path graphs $P_m o P_n$ with pendant vertex as root and determine the unidomination number of $P_m o P_n$.

KEYWORDS: Rooted Product graph, Unidominating functions, Unidomination number.

Subject Classification: 68R10

1.INTRODUCTION

Unidominating functions was introduced by V. Anantha Lakshmi and B. Maheshwari [15]in 2015, where they presented unidominating function for path graph. In this paper we studied unidominating function for rooted product of two path graph P_m and P_n with pendent vertex as root. We find the unidomination number of $P_m o P_n$ and then determine the number of unidominating function of minimum weight for $P_m o P_n$

Definition 1.1: The rooted product of two graphs of G_1 and G_2 denoted by $G_1 \circ G_2$, is the graph obtained by choosing one vertex of G_2 as root and then attaching the root vertex of copy of G_2 to each of the vertices of G_1.

For the rooted product of two path graphs P_m with P_n. Let V(P_m) = { $v_1, v_2, v_3, \dots \dots \dots \dots . v_m$} and $V(P_n) = \{u_1, u_2, u_3, \dots \dots \dots u_n\}$ be the vertex sets of P_m and P_n respectively. Let the root vertex chosen from P_n be the pendant vertex u_1. So the root vertex set of $P_m \circ P_n$ with m-copies of P_n becomes

$\{ (u_1, v_1), (u_1, v_2), \dots \dots \dots \dots \dots . (u_1, v_m)\}$

Definition 1.2: Let G(V,E) be a graph. A function $f: V \to \{0,1\}$ is said to be a unidominating function

If $\sum_{u\in N[v]} f(u) \geq 1$ and $f(v) = 1$

$\sum_{u\in N[v]} f(u) = 1$ and $f(v) = 0$

$f(V) = \sum_{u\in v} f(u)$ is called the weight of the function f and is denoted by $\gamma_u(G)$.

Definition 1.3 : The uni domination number of a graph G (V,E) is

$\gamma_u(G)$ = min $\{f(V)/f \text{ is a uni dominating function } f \text{ on } G\}$

1.UNIDOMINATING FUNCTION OF $P_m o P_n$

In this section we find the unidominating function of minimum weight on $P_m o P_n$ and hence find the unidomination number.

Theorem 2.1: The Unidomination number of rooted product of $P_m o P_n$ with pendant vertex as root is

$$\gamma_u(P_m o P_n) = \begin{cases} X + 2a + 1 + k & \text{for m} \equiv 2 \text{ (mod3), n} \equiv 2 \text{ (mod3)} \\ X + r_1 a + \left\lfloor \frac{r_1}{2} \right\rfloor + 2\left\lfloor \frac{r_2}{2} \right\rfloor k & \text{for m} \equiv 0,1 \text{ (mod3), n} \equiv 0,1 \text{ (mod3)} \end{cases}$$

Where m= 3k+r_1, n = 3a+r_2 ,$X = k(3a + 1)$

Proof: Consider the rooted product graph $P_m o P_n$.Let the vertex set of P_m be V(P_m) = $\{v_1, v_2, v_3, \dots \dots \dots v_m\}$ and vertex set of P_n be V(P_n) = $\{u_1, u_2, u_3, \dots \dots .. u_n\}$ in the rooted product $P_m o P_n$. Let the root vertex be the pendent vertex $u_1$of P_n identified with jth vertex of v_j of P_m, so

V ($P_m o P_n$) = $\{(u_i, v_j) : \ i = 1 \text{ to } n ,\ j = 1 \text{ to } m\}$

The unidomination number of P_n is based on the following minimum weight function for a path $P_n$as

$$\begin{cases} 1 & \text{for n} \equiv 0\text{(mod3)} \\ \left\lceil \frac{n}{3} \right\rceil & \text{for n} \equiv 1 \text{ (mod3)} \\ 2 & \text{for n} \equiv 2 \text{ (mod3)} \end{cases}$$

We use this result proved by V.Anantha Lakshmi and B.Maheshwari in [15] . We extend the function definition for the root vertex in $P_m o P_n$ as

$$f(u_1, v_j) = \begin{cases} 1 & \text{for j} \equiv 2\text{(mod3)} \\ 0 & \text{for j} \equiv 0{,}1\text{(mod3)} \end{cases}$$

As there are m-copies of P_n from [15] we get, $\gamma_u(P_m o P_n) \geq m\gamma_u(P_n)$ ------------- (1)

but as these m-paths have adjacency between root vertices we need to check for the minimal function value.

For any vertex $f(u_i, v_j) = 1$

$\sum_{(x,y)\epsilon N[u_i,v_j]} f(x,y) \geq 1$ is natural as $f(u_i, v_j) = 1$

For $f(u_i, v_j) = 0$ then

$$\sum_{(x,y)\epsilon N[u_i,v_j]} f(x,y) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j) = 1$$

$$\text{Or } f(u_i, v_{j-1}) + f(u_i, v_j) + f(u_i, v_{j+1}) = 1$$

For unidominating condition to be satisfied it is essential that exactly one of $f(u_{i-1}, v_j), f(u_{i+1}, v_j)$ should be equal to one.

Therefore we need to check unidominating condition for only those for which $f(u_i, v_j) = 0$

Case(i): For $m \equiv 0(mod3)$

For m=3k, the 2k vertices are $(u_1, v_1), (u_1, v_3)(u_1, v_4)(u_1, v_6)$ … … … .. are assigned the function value zero, the remaining k- vertices $(u_1, v_2), (u_1, v_5)(u_1, v_8)$…….. are assigned the function value one. For the copy of P_n attached with these vertices, (n-1) path vertices $(u_2, v_j)(u_3, v_j)$ … … …. (u_n, v_j) at the vertex (u_1, v_j) for j= 1,2,3……m as follows

We define the function value as follows.

Subcase (IA): For n≡ 0 (mod3) let n =3a

For the m-copies of P_n we define the function value as ,

For j≡ 0,1(mod3)

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For j≡2(mod3)

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

(i) For j$\not\equiv 2(mod3)$ when $f(u_i, v_j) = 0$

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$$f(u_i, v_j)=\begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

(ii) For $j\equiv 2(mod3)$, when $f(u_i, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

and $f(u_{n-1}, v_j) = 1$

For $f(u_i, v_j) = \begin{cases} 0 & \text{for } j \equiv 0 \; or \; 1(\text{mod}3) and \; i \equiv 1, or \; 2(\text{mod}3) \\ 1 & \text{for } j \equiv 2(\text{mod}3) and \; i \equiv 0 \; or \; 2(\text{mod}3) \end{cases}$

To check the unidomination condition for function f at vertices when $f(u_i, v_j) = 0$

Case (a): For $j\equiv 0(\text{mod}3)$, $i\equiv 1(\text{mod}3)$

1. $f(u_1, v_j) = f(u_1, v_{j-1}) + f(u_1, v_j) + f(u_1, v_{j+1}) + f(u_2, v_j)$ = 1+0+0+0=1
2. $f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + f(u_2, v_m)$ = 1+0+0=1
3. $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$= 1+0+0=1
4. $f(u_i, v_m)$ =$f(u_{i-1}, v_m)$ +$f(u_i, v_m) + f(u_{i+1}, v_m)$= 1+0+0=1

Case (b): For $j\equiv 0(mod3)$, $i \equiv 2(mod3)$

5. $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$= 0+0+1=1

Case (c): For $j\equiv 1(mod3)$, $i \equiv 1(mod3)$

6. $f(u_1, v_1)$ =$f(u_2, v_1)$ +$f(u_1, v_1)$ +$f(u_1, v_2)$ =0+0+1=1
7. $f(u_i, v_1)$ =$f(u_{i+1}, v_1)$ +$f(u_i, v_1)$ +$f(u_i, v_2)$ =0+0+1=1
8. $f(u_1, v_j)$ =$f(u_1, v_{j-1})$ +$f(u_1, v_j)$ +$f(u_1, v_{j+1})$ +$f(u_2, v_j)$ =0+0+1+0=1
9. $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$ = 1+0+0=1

Case (d): For $j\equiv 1(mod3)$, $i \equiv 2(mod3)$

10. $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$ =0+0+1=1

Case (e): For $j\equiv 2(mod3)$, $i \equiv 0(mod3)$

11. $f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$ =0+0+1=1
12. $f(u_n, v_j) = f(u_{n-1}, v_j)$ +$f(u_n, v_j)$= 1+0=1

Case (f): For $j\equiv 2(mod3)$, $i \equiv 2(mod3)$

13.$f(u_i, v_j) = f(u_{i-1}, v_j) + f(u_i, v_j) + f(u_{i+1}, v_j)$ =1+0+0=1

As at all 13 cases above when $f(u_i, v_j) = 0,\ \sum_{u \in N} f = 1$ we get that for $m \equiv 0(mod3)$ and $n \equiv 0(mod3)$ the function f satisfies unidomination condition with weight of the function f(V) equal to

$$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$$

= 2k(0+0+1+0+0+1+-----------------0+0+1)

+k(1+0+0+1+0+0+----------------+0+1+1+0)

$$= 2k\left(\frac{n}{3}\right) + k[\left(\frac{n}{3}\right) + 1]$$

$$= 2\left[\frac{m}{3}\right](a) + \left[\frac{m}{3}\right](a+1)$$

Subcase (IB): n≡ 1 (mod3) let n = 3a+1

For j$\not\equiv 2(mod3)$ when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For j$\equiv 2(mod3)$ when $f(u_1, v_j) = 1$

$$f(u_i, v_j = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

The function definition is identical to the definition given in subcase IA except at values i= n-1, n-2 for j$\equiv 2(mod3)$. Therefore we check the unidomination condition only for these two values of i

$f(u_{n-2}, v_j) = f(u_{n-3}, v_j) + f(u_{n-2}, v_j) + f(u_{n-1}, v_j)$ = 1+0+0 =1

$f(u_{n-1}, v_j) = f(u_{n-2}, v_j) + f(u_{n-1}, v_j) + f(u_n, v_j)$ = 0+0+1 =1

Hence the uni domination condition satisfiedfor all (u_i, v_j) with weight of the function as,

$$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$$

= 2k(0+0+1+0+0+1+-----------------0+0+1+0)

+k(1+0+0+1+0+0+----------------+1+0+0+1)

$$= 2k\left[\frac{n}{3}\right] + k(\left[\frac{n}{3}\right] + 1)$$

$$= 2\left[\frac{m}{3}\right](a) + \left[\frac{m}{3}\right](a+1)$$

Subcase (IC): n≡ 2 (mod3) let n=3a+2

For j$\not\equiv 2(mod3)$ when $f(u_1, v_j) = 0$

$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$

For j$\equiv 2(mod3)$ when $f(u_1, v_j) = 1$

$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$

As the function definition is identical to the definition given in subcase IB except at four values i= n, n-1, n-2, n-3 for j $\equiv 2(\text{mod}3)$Therefore we check the unidomination condition for j$\not\equiv 2(mod3)$ and i = n , n-3 when the functional value is zero.

$f(u_n, v_j) = f(u_{n-1}, v_j) + f(u_n, v_j)$= 1+0 =1

$f(u_{n-3}, v_j) = f(u_{n-4}, v_j) + f(u_{n-3}, v_j) + f(u_{n-2}, v_j)$= 0+0+1 =1

Hence the uni domination condition is satisfied with weight of the function.

$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= 2k(0+0+1+0+0+1+-----------------0+1+1+0)

+k(1+0+0+1+0+0+----------------+1+0+0+1)

$= 2k\left(\left[\frac{n}{3}\right] + 1\right) + k\left(\left[\frac{n}{3}\right] + 1\right)$

= 2k (a+1) +k (a+1)

Case (II): For m=1 (mod3) let m= 3k +1

Subcase (IIA): n$\equiv$ 0 (mod3) let n =3a

For j$\not\equiv 2(mod3)$ and j$\neq$ n-1 when $f(u_1, v_j) = 0$

$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$

For j$\equiv 2(mod3)$ and j= n-1 when $f(u_1, v_j) = 1$

$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$

The function definition is identical to the definition given in subcase (IA) except at values j = m, m-1,m-2, m-3 for i=1. Therefore we check only the unidomination condition for these values

i = 1, j =m , m-3 only when the function value is zero.

$f(u_1, v_{m-3}) = f(u_1, v_{m-4}) + f(u_1, v_{m-3}) + f(u_1, v_{m-2}) + f(u_1, v_{m-3})$ = 0+0+1+0 = 1

$f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + f(u_2, v_m) = 1 + 0 + 0 = 1$

Hence the unidomination condition is satisfied with weight of the function.

$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

$= 2k\,(0+0+1+0+0+1+\ldots\ldots+0+0+1)$

$+ (k+1)(1+0+0+1+0+0+\ldots\ldots+0+1+1+0)$

$= 2k[\frac{n}{3}] + (k+1)\left(\left[\frac{n}{3}\right] + 1\right)$

$= 2k(a) + (k+1)(a+1)$

Subcase (II B): n≡ 1 (mod3) let n= 3a+1

For $j \not\equiv 2(mod 3)$ when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For $j \equiv 2(mod 3)$ and j= n-1, when $f(u_1, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

The function definition is identical to the definition given in subcase IA except at values n-1, n-2 for i=1. Therefore we check the unidomination condition for these four values i for all $j \equiv 2(mod 3)$ when the function value is zero.

$f(u_{n-2}, v_j) = f(u_{n-3}, v_j) + f(u_{n-2}, v_j) + f(u_{n-1}, v_j) = 0+0+1 = 1$

$f(u_{n-1}, v_j) = f(u_{n-2}, v_j) + f(u_{n-1}, v_j) + f(u_n, v_j) = 0+1+0 = 1$

Hence the unidomination condition is satisfied with weight of the function.

$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

$= 2k\,(0+0+1+0+0+1+\ldots\ldots+0+0+1+0)$

$+ (k+1)(1+0+0+1+0+0+\ldots\ldots+1+0+0+1)$

$= 2k[\frac{n}{3}] + (k+1)\left(\left[\frac{n}{3}\right] + 1\right)$

$= 2k(a) + (k+1)(a+1)$

Subcase(II C): n≡ 2 (mod3) let n= 3a+2

For $j \not\equiv 2(mod 3)$ when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For $j \equiv 2(mod 3)$ and j= n-1,when $f(u_1, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

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The function definition is identical to the definition given in the subcase II B except at values i = n , n-1, n-2, n-3 for j$\not\equiv 2(mod3)$. Therefore we check the unidomination condition for j$\not\equiv 2(mod3)$ and i = n, n-3 when the function value is zero.

$f(u_n, v_j) = f(u_{n-1}, v_j) + f(u_n, v_j) = 1+0 = 1$

$f(u_{n-3}, v_j) = f(u_{n-4}, v_j) + f(u_{n-3}, v_j) + f(u_{n-2}, v_j) = 0+0+1 = 1$

Hence the unidomination condition is satisfied with weight of the function.

$$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$$

$$= 2k\,(0+0+1+0+0+1+\ldots\ldots+0+1+1+0)$$

$$+ (k+1)(1+0+0+1+0+0+\ldots\ldots+1+0+0+1+0)$$

$$= 2k\,([\tfrac{n}{3}] + 1) + (k+1)\,(\left[\tfrac{n}{3}\right] + 1)$$

$$= 2k\,(a+1) + (k+1)(a+1)$$

Case (III): For m=2 (mod3) let m= 3k +2

Subcase (IIIA): n≡ 0 (mod3) let n =3a

For j$\not\equiv 2(mod3)$,when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For j$\equiv 2(mod3)$ and j= n-1,when $f(u_1, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

The function definition is identical to the definition given in subcase (IA) except at values j = m , m-1, m-2, m-3 for i=1. Therefore we check the unidomination condition for these four values i=1, j=m, m-2 only when the function value is zero.

$f(u_1, v_{m-3}) = f(u_1, v_{m-4}) + f(u_1, v_{m-3}) + f(u_1, v_{m-2}) + f(u_2, v_{m-3}) = 0+1+0+0 = 1$

$f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + f(u_2, v_m) = 1 + 0 + 0 = 1$

Hence the uni domination condition is satisfied with weight of the function.

$$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$$

$$= (2k+1)\,(0+0+1+0+0+1+\ldots\ldots+0+0+1)$$

$$+ (k+1)(1+0+0+1+0+0+\ldots\ldots+0+1+1+0)$$

$$=(2k+1)([\tfrac{n}{3}]) + (k+1)\,([\tfrac{n}{3}]+1)$$

$$=(2k+1)(a)+(k+1)(a+1)$$

Subcase (IIIB): n≡ 1 (mod3)

For $j \not\equiv 2(mod3)$, when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For $j \equiv 2(mod3)$ and j= n-1, when $f(u_1, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

The function definition is identical to the definition given in subcase IA except at values j= m , m-1, m-2, m-3 for i=1. Therefore we check the unidomination condition for these values i=1, j=m,m-2 only when the function value is zero.

$f(u_1, v_{m-3}) = f(u_1, v_{m-4}) + f(u_1, v_{m-3}) + f(u_1, v_{m-2}) + f(u_2, v_{m-3})$ = 0+1+0+0 = 1

$f(u_1, v_m) = f(u_1, v_{m-1}) + f(u_1, v_m) + f(u_2, v_m) = 1 + 0 + 0 = 1$

Hence the uni domination condition is satisfied with weight of the function.

$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$

= (2k+1) (0+0+1+0+0+1+…………………+0+0+1+0)

+ (k+1)(1+0+0+1+0+0+……………..+1+0+0+1)

$=(2k+1)([\frac{n}{3}])+ (k+1) ([\frac{n}{3}]+1)$

=(2k+1)(a)+(k+1)(a+1)

From all the above cases we can combine the equation into one common expression for function value as,

$$\boldsymbol{f(V) = X + r_1 a + \left\lceil \frac{r_1}{2} \right\rceil + 2 \left\lfloor \frac{r_2}{2} \right\rfloor k}$$

Where m= 3k+r_1, n = 3a+r_2 , X= k(3a+1)

Subcase (IIIC): n≡ 2 (mod3)

For $j \not\equiv 2(mod3)$, when $f(u_1, v_j) = 0$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 0(\text{mod}3) \\ 0 & \text{for } i \equiv 1,2(\text{mod}3) \end{cases}$$

For $j \equiv 2(mod3)$ and j= n-1,when $f(u_1, v_j) = 1$

$$f(u_i, v_j) = \begin{cases} 1 & \text{for } i \equiv 1(\text{mod}3) \\ 0 & \text{for } i \equiv 0,2(\text{mod}3) \text{and } i \neq n-1 \end{cases}$$

Hence the unidomination condition is satisfied with weight of the function.

$$f(V) = \sum_{i=1}^{n} \sum_{j=1}^{m} f(u_i, v_j)$$

$$= (2k+1)(0+0+1+0+0+1+\ldots\ldots+0+1+1+0)$$

$$+ (k+1)(1+0+0+1+0+0+\ldots\ldots+0+0+1+0)$$

$$=(2k+1)\left(\left[\tfrac{n}{3}\right]+1\right) + (k+1)\left[\tfrac{n}{3}\right]$$

$$=(2k+1)(a+1)+(k+1)(a)$$

For last case we can write

f(V) = X+2a+1+k

Where m= 3k+r_1, n = 3a+r_2 , X= k(3a+1)

	m=3k	m=3k+1	m=3k+2
n=3a	2k(a)+k(a+1)	2k(a)+(k+1)(a+1)	(2k+1)(a)+(k+1)(a+1)
n=3a+1	2k(a)+k(a+1)	2k(a)+(k+1)(a+1)	(2k+1)(a)+(k+1)(a+1)
n=3a+2	2k(a+1)+k(a+1)	2k(a+1)+(k+1)(a+1)	(2k+1)(a+1)+(k+1)(a)

Using the minimality of the function definition on path graph [15] and equation (1) state the function has minimal weight f(V)

Hence combining all the three cases we get unidomination number of rooted product of $P_m o P_n$ is

$$\gamma_u(P_m o P_n) =$$

$$\begin{cases} X + 2a + 1 + k & \text{for } m \equiv 2 \ (\text{mod}3), n \equiv 2 \ (\text{mod}3) \\ X + r_1 a + \left\lfloor \frac{r_1}{2} \right\rfloor + 2\left\lfloor \frac{r_2}{2} \right\rfloor k & \text{for } m \equiv 0,1 \ (\text{mod}3), n \equiv 0,1 \ (\text{mod}3) \end{cases}$$

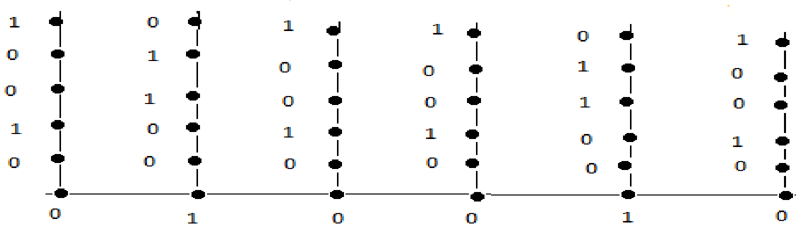
Where m= 3k+r_1, n = 3a+r_2, $X = k(3a+1)$, ,

3. ILLUSTRATIONS :

Example 3.1: Let $m = 6, n = 6$

Clearly $6 \equiv 0(mod3)$.

The functional values of a unidominating function f defined in case I and subcase IA of theorem 2.1 are given at the corresponding vertices of $P_m o P_n$



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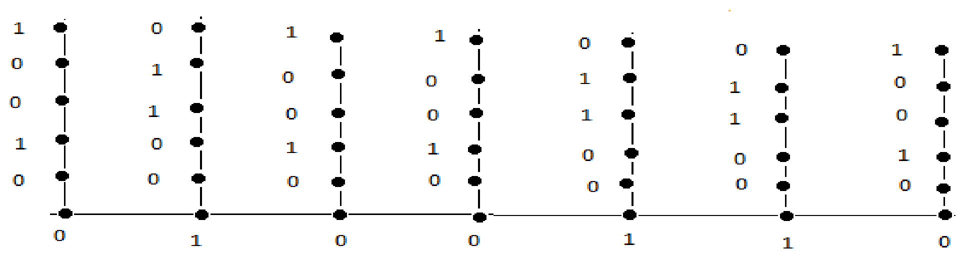
Unidomination number of $P_6 o P_6$is $\gamma_u(P_6 o P_6) = 14$

Example 3.2: Let $m = 7$, $n = 6$

Clearly $7 \equiv 1(mod 3)$

The functional values of a unidominating function f defined in case II and subcase IIA of theorem 2.1 are given at the corresponding vertices of $P_7 o P_6$

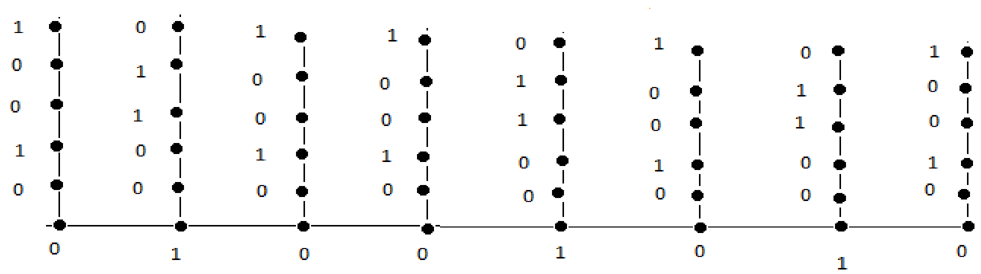


Unidomination number of $P_7 o P_6$ is $\gamma_u(P_7 o P_6) = 17$

Example 3. 3: Let $m = 8$, $n = 6$

Clearly $8 \equiv 2(mod 3)$

The functional values of a unidominating function f defined in case III and subcase IIIA of theorem 2.1 are given at the corresponding vertices of $P_8 o P_6$



REFERENCES:

1. J.A. Bonday, U.S.R. Murty, Graph theory with applications, Macmillan Press, London ,1976.
2. B. Chaffin, J.P.Linderman, N.J.A. Sloane, A.R. Wilks, On curling numbers of integer sequences, J.Integer Seq.,16(2013),Article-13.4.3,1-31.
3. G. Chartrand, L.Lesniak, Graphs and digraphs ,CRC Press,2000.
4. Godsil C.D., Mckay B.D., a new graph product and its spectrum, Bulletin of the Australlian mathematical society 18(1) (1978) 21-28.

5. J.T.Gross, J.Yellen, Graph theory and its applications, CRC Press,2006
6. R.Hammack, W.Imrich and S.Klavzar, Handbook of product graphs, CRC Press,2011.
7. F.Harary, Graph theory ,New Age International, Delhi.,2001
8. Haynes T.W, Hedetniemi S.T, Slater P.J, Fundementals of domination in graphs , Marcel Dekker , Inc. New York ,1998 .
9. W. Imrich, S.Klavzar, Product graphs: Structure and recognition,Wiley,2000.
10. Ore O , Theory of graphs , Amer . Math .Soc. Collaq. Pub., 38 (1962)
11. Rashmi S B, Dr. Indrani Pramod Kelkar, Domination number of Rooted product graph $P_m \odot C_n$, Journal of computer and Mathematical Sciences ,Vol.7(9),469-471, September 2016.
12. Rashmi S B, Dr. Indrani Pramod Kelkar, Total Domination number of Rooted product graph $P_m \odot C_n$,International Journal of Advanced Research in Computer science, Volume 8, No.6, July 2017(Special Issue) .zzx
13. Rashmi S B, Dr. Indrani Pramod Kelkar, Signed domination number of rooted product of a path with cycle graph , International Journal of Mathematical Trends and Techonology, Volume 58, Issue 1-June 2018.
14. Rashmi S B, Dr. Indrani Pramod Kelkar, Rajanna K R, Signed and Total Signed dominating function of $P_m \odot S_{n+1}$,International Journal of Pure and Applied Mathematics, Volume 119, No.14 2018, 193-197.
15. V. Anantha Lakshmi and B. Maheshwari , Unidominating functions of a path , International Journal of Computer Engineering & Technology , Volume 6, Issue 8 , Aug 2015 , pp.11-19.