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TRIPOLAR FUZZY WEAK BI-IDEALS OF A GAMMA NEAR ALGEBRA

RAKSHITA DESHMUKH[1]
P. NARASIMHA SWAMY[2]
B. JYOTHI[3]

[1,3] *Department of Mathematics, University College for Women, Osmania University, Hyderabad, Telangana, INDIA.*
[2] *Department of Mathematics, GITAM University, Hyderabad Campus, Telangana, INDIA.*

ABSTRACT

Based on the concept of fuzzy set theory,here we introduce the concept of tripolar fuzzy weak bi ideals of a gamma near algebra.We prove some of its properties.We have proved that the homomorphic image of a tripolar fuzzy weak bi ideal of a gamma near algebra is also a tripolar fuzzy weak bi ideal.We have also proved that the cartesian product of tripolar fuzzy weak bi ideals of a gamma near algebra is also a tripolar fuzzy weak bi ideal.

KEYWORDS:

Tripolar Fuzzy Near Algebra;
Tripolar Fuzzy Gamma Near Algebra;
Tripolar Fuzzy Weak Bi-ideal.

1. INTRODUCTION

The concept of gamma ring (a generalization of a ring) was introduced by Nobusawa[9] and generalized by Barnes[2]. A generalization of both the concepts gamma ring and near ring namely gamma near ring was introduced by Bh.Satyanarayana[11]. Brown, Yamamuro, Irish and Srinivas have studied the structure of near algebra. Gamma near algebra is a generalization of both the concepts of near algebra and gamma near ring is introduced by Srinivas and P. Narasimha swamy[12].

The theory of fuzzy sets is most appropriate theory for dealing with uncertainty was introduced by L.A.Zadeh [13]. There are many extensions of fuzzy sets like intuitionistic fuzzy set, interval valued fuzzy sets, vague sets, bipolar fuzzy sets etc.

The concept of weak bi-ideals of a near ring was studied by Chinnadurai and Kadalarasi[4]. K.T.Attenesov[1]studied intuitionistic fuzzy sets. K.M.Lee[7] introduced the concept of bipolar fuzzy set. Bipolar fuzzy sets are an extension of fuzzy sets whose memberely degree range is [-1,1] . Hyoung Gu Baik [5] has studied the concept of bipolar fuzzy ideals of near rings.

The notion of tripolar fuzzy set is to be able to deal with tripolar information as a generalization of fuzzy set, bi polar fuzzy set, intuitionistic fuzzy set. The tripolar fuzzy set is very useful in discriminating relevant elements, irrelevant elements and contrary elements. Tripolar fuzzy interior ideals of a gamma semiring was studied by M. Murali Krishna Rao and B. Venkateshwarlu [8].

Here we intend to study the concept of tripolar fuzzy weak bi-ideals of a gamma near algebra. We study some of their properties.

2. PRELIMINARIES

Definition 2.1: Let *Y* be a linear space over a field *X* and Γ be a non-empty set. Then *Y* is said to be a Γ-near-algebra over a field *X*, if there exist a mapping $Y \times \Gamma \times Y \to Y$. The image of (a, α, b) is denoted by $(a\alpha b)$ satisfying the following conditions:

i) $(a\alpha b)\beta c = a\alpha(b\beta c)$,
ii) $(a+b)\alpha c = a\alpha c + b\alpha c$,
iii) $(\lambda a)\alpha b = \lambda(a\alpha b)$ for every $a, b, c \in Y$, $\alpha, \beta \in \Gamma$ and $\lambda \in X$.

Definition 2.2: Let *Y* be a Γ-near-algebra over a field *X*. Then *Y* is said to be a zero symmetric Γ-near-algebra or Γ-near-c-algebra if $a\alpha b = 0$ for every $a \in Y$ and $\alpha \in \Gamma$, where 0 is the additive identity in *Y*.

Definition 2.3: Let *Y* be a Γ-near-algebra over a field *X*. A linear sub-space *L* of *Y* is said to be a sub Γ-near-algebra over a field *X*, if there exist a mapping $L \times \Gamma \times L \to L$ satisfying the following three conditions:

i) $(a\alpha b)\beta c = a\alpha(b\beta c)$,
ii) $(a+b)\alpha c = a\alpha c + b\alpha c$,
iii) $(\lambda a)\alpha b = \lambda(a\alpha b)$ for every $a, b, c \in L, \alpha, \beta \in \Gamma$ and $\lambda \in X$.

Definition 2.4: Let *Y* be Γ-near-algebra over a field *X*. Then the linear sub-space *I* of the linear space *Y* is called

i) A left ideal if $y\alpha(x+i) - y\alpha x \in I$ for all $x, y \in Y, \alpha \in \Gamma, i \in I$,
ii) A right ideal if $i\alpha x \in I$ for all $x \in Y, \alpha \in \Gamma, i \in I$.

Definition 2.5: Let *N* and N^1 be two Γ-near-algebra over a field *X*. A mapping $\phi : N \to N^1$ is called a Γ-near-algebra homomorphism if the following three conditions hold:

i) $\phi(x+y) = \phi(x) + \phi(y)$,
ii) $\phi(\lambda x) = \lambda\phi(x)$,
iii) $\phi(x\alpha y) = \phi(x)\alpha\phi(y)$ for every $x, y \in N, \lambda \in X, \alpha \in \Gamma$.

Definition 2.6: Let *N* and N^1 be two Γ-near-algebra over a field *X*. Let $\phi : N \to N^1$ be a Γ-near-algebra homomorphism. Then the homomorphic image of *N* is $\phi(N)$ defined by $\phi(N) = (x^1 \in N^1 : \phi(x) = x^1, x \in N)$.

Definition 2.7: A fuzzy set *T* of a universe set *X* is said to be a tripolar fuzzy set, if $T = ((x, \alpha_T(x), \beta_T(x), \gamma_T(x)) / x \in X$ and $0 \le \alpha_T(x) + \beta_T(x) \le 1)$.
where $\alpha_T : X \to [0,1], \beta_T : X \to [0,1], \gamma_T : X \to [-1,0]$.

The membership degree $\alpha_T(x)$ characterizes the extent that the element *x* satisfies to the property corresponding to tripolar fuzzy set *T*, $\beta_T(x)$ characterizes the extent that the element *x* satisfies to the not property (irrelevant) corresponding to tripolar fuzzy set *A* and $\gamma_T(x)$ characterizes the extent that the element *x* satisfies to the implicit counter property of tripolar fuzzy set *T*.

We write $T=(\alpha_T,\beta_T,\gamma_T)$ for
$T=((x,\alpha_T(x),\beta_T(x),\gamma_T(x))/x\in X$ and $0\le\alpha_T(x)+\beta_T(x)\le 1)$.

Thus, a tripolar fuzzy set *T* is a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set.

3. TRIPOLAR FUZZY WEAK BI-IDEALS OF A GAMMA NEAR ALGEBRA

In this section, we introduce the concept of tripolar fuzzy weak bi-ideals of a gamma near algebra. We also study some of their properties.

Definition 3.1: Let *X* be a field. A tripolar fuzzy set $F=(\alpha_F,\beta_F,\gamma_F)$ of *X* is called a tripolar fuzzy field of *X* if it satisfies the following four conditions for every $p,q\in X$

i) $\alpha_F(p+q)\ge\min(\alpha_F(p),\alpha_F(q))$, $\beta_F(p+q)\le\max(\beta_F(p),\beta_F(q))$ and $\gamma_F(p+q)\le(\gamma_F(p),\gamma_F(q))$.
ii) $\alpha_F(-p)\ge\alpha_F(p)$, $\beta_F(-p)\le\beta_F(p)$ and $\gamma_F(-p)\le\gamma_F(p)$.
iii) $\alpha_F(pq)\ge\min(\alpha_F(p),\alpha_F(q))$, $\beta_F(pq)\le\max(\beta_F(p),\beta_F(q))$ and $\gamma_F(pq)\le\max(\gamma_F(p),\gamma_F(q))$.
iv) $\alpha_F(p^{-1})\ge\alpha_F(p)$, $\beta_F(p^{-1})\le\beta_F(p)$ and $\gamma_F(p^{-1})\le\gamma_F(p)$ for $p(\ne 0)\in X$.

Definition 3.2: Let *Y* be a gamma near algebra over a field *X*. A tripolar fuzzy set $T=(\alpha_T,\beta_T,\gamma_T)$ of *Y* is called a tripolar fuzzy gamma near algebra over a tripolar fuzzy field $F=(\alpha_F,\beta_F,\gamma_F)$ of *X* if it satisfies the following four conditions:

i) $\alpha_T(p+q)\ge\min(\alpha_T(p),\alpha_T(q))$, $\beta_T(p+q)\le\max(\beta_T(p),\beta_T(q))$ and $\gamma_T(p+q)\le(\gamma_T(p),\gamma_T(q))$.
ii) $\alpha_T(\lambda p)\ge\min(\alpha_F(\lambda),\alpha_T(p))$, $\beta_T(\lambda p)\le\max(\beta_F(\lambda),\beta_T(p))$ and $\gamma_T(\lambda p)\le\max(\gamma_F(\lambda),\gamma_T(p))$.
iii) $\alpha_T(p\eta q)\ge\min(\alpha_T(p),\alpha_T(q))$, $\beta_T(p\eta q)\le\max(\beta_T(p),\beta_T(q))$ and $\gamma_T(p\eta q)\le\max(\gamma_T(p),\gamma_T(q))$.
iv) $\alpha_F(1)\ge\alpha_T(p)$, $\beta_F(1)\le\beta_T(p)$ and $\gamma_F(1)\le\gamma_T(p)$ for every $p,q\in Y$, $\eta\in\Gamma,\lambda\in X$, 1 is the unity in *X*.

Example 3.4: Let $Y=\{0,a,b,c\}$ be a set with binary operation "+" and "." as follows:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	0	b	b
c	0	0	c	c

Let $X=Z_2=\{0,1\}_{\oplus_2,\otimes_2}$ be a field.

Let $\Gamma=(\eta,\nu)$ be a non-empty set. Define a mapping $Y\times\Gamma\times Y\to Y$ by the following tables:

μ	0	a	b	c
0	0	0	0	0

δ	0	a	b	c
0	0	0	0	0

a	0	0	0	0
b	0	0	0	0
c	0	0	0	0

a	0	0	*a*	*a*
b	0	0	*b*	*b*
c	0	0	*c*	*c*

Let $F=(\alpha_F,\beta_F,\gamma_F)=(x\in X;\alpha_F(x),\beta_F(x),\gamma_F(x))$ be a tripolar fuzzy set of *X* defined as $F=\{(0;0.9,0.1,-0.9),(1;0.8,0.2,-0.8)\}$. Then *F* is a tripolar fuzzy field of *X*.

Let a scalar multiplication on *Y* is defined by $0.x=0, 1.x=x$ for every $x\in Y; 0,1\in X$. Then *Y* is a near algebra over a field *X*.

Let $T=(\alpha_T,\beta_T,\gamma_T)=(y\in Y;\alpha_T(y),\beta_T(y),\gamma_T(y))$ be tripolar fuzzy set of *Y*.

defined as $T=\{(0;0.6,0.4.-0.6),(a;0.4,0.6,-0.4),(b;0.4,0.6,-0.4),(c;0.4,0.6,-0.4)\}$.

Then *T* is a tripolar fuzzy weak bi-ideal of a gamma near algebra *Y*.

Theorem 3.5: If a tripolar fuzzy set $T=(\alpha_T,\beta_T,\gamma_T)$ of a gamma near algebra *Y* over fuzzy field $F=(\alpha_F,\beta_F,\gamma_F)$ of *X* is a tripolar fuzzy weak bi-ideal ideal of a gamma near algebra *Y* then

$T=(\alpha_T,\bar{\alpha}_T,\gamma_T)$ is a tripolar fuzzy weak bi-ideal of a gamma near algebra *Y*.

Proof: Let $p,q\in Y,\eta,\nu\in\Gamma$ then

$$\bar{\alpha}_T(p+q)=1-\alpha_T(p+q)\le 1-\min(\alpha_T(p),\alpha_T(q))\le\max(1-\alpha_T(p),1-\alpha_T(q))=\max(\bar{\alpha}_T(p),\bar{\alpha}_T(q)).$$

$$\bar{\alpha}_T((p\eta q\nu r))=1\text{-}\alpha_T((p\eta q\nu r))\le 1-\min(\alpha_T(p\eta q\nu r))\le\max(1-(\alpha_T(p),\alpha_T(q),\alpha_T(r))$$

$$\le\max(1\text{-}\alpha_T(p),1-\alpha_T(q),1-\alpha_T(r))=\max(\bar{\alpha}_T(p),\bar{\alpha}_T(q),\bar{\alpha}_T(r)).$$

$$\bar{\alpha}_T(\lambda p)=1-\alpha_T(\lambda p)\le 1-\min(\alpha_F(\lambda),\alpha_T(p))\le\max(\alpha_F(\lambda),1-\alpha_T(p))=\max(\alpha_F(\lambda),\bar{\alpha}_T(p)).$$

$$\bar{\alpha}_F(1)=1-\alpha_F(1)\le 1-\min(\alpha_F(1))\le\max(1-\alpha_F(1))=\bar{\alpha}_T(p).$$

Thus $(\alpha_T,\bar{\alpha}_T,\gamma_T)$ is a tripolar fuzzy weak bi- ideal of a gamma near algebra *Y*.

Theorem 3.6: A tripolar fuzzy set $T=(\alpha_T,\beta_T,\gamma_T)$ is a fuzzy weak bi- ideal of a gamma near algebra *Y* over a tripolar fuzzy field $F=(\alpha_F,\beta_F,\gamma_F)$ in *X* if fuzzy subsets $(\bar{\beta}_T,\beta_T,\gamma_T)$ are fuzzy weak bi-ideals of a gamma near algebra *Y*.

Proof: Suppose $T=(\alpha_T,\beta_T,\gamma_T)$ is a fuzzy weak bi-ideal of a gamma near algebra *Y*. Then obviously, α_T,γ_T are fuzzy weak bi-ideals of *Y*. Now,

$$\bar{\beta}_T(p+q)=1-\beta_T(p+q)\ge 1-\max(\beta_T(p),\beta_T(q))\ge\min(1-\beta_T(p),1-\beta_T(q))=\min(\bar{\beta}_T(p),\bar{\beta}_T(q)).$$

$$\bar{\beta}_T(p\eta q\nu r)=1-\beta_T(p\eta q\nu r)\ge 1-\max(\beta_T(p),\beta_T(q),\beta_T(r))\ge\min(1-(\beta_T(p),\beta_T(q),\beta_T(r))$$

$$=\min(1-\bar{\beta}_T(p),1-\bar{\beta}_T(q),1-\bar{\beta}_T(r))=\min(\bar{\beta}_T(p),\bar{\beta}_T(q),\bar{\beta}_T(r)).$$

$$\bar{\beta}_T(\lambda p)=1-\beta_T(\lambda p)\ge 1-\max(\beta_F(\lambda),\beta_T(p))\ge\min(\beta_F(\lambda),1-\beta_T(p))=\min(\beta_F(\lambda),\bar{\beta}_T(p)).$$

$$\bar{\beta}_F(1)=1-\beta_F(1)\ge 1-\max(\beta_F(1))\ge\max(1-\beta_F(1))=\bar{\beta}_T(p).$$

Thus $(\bar{\beta}_T,\beta_T,\gamma_T)$ is a tripolar fuzzy weak bi- ideal of a gamma near algebra *Y*.

Theorem 3.7: Let $f:M\to N$ be a homomorphism of a gamma near algebras. If $T=(\alpha_T,\beta_T,\gamma_T)$ is a tripolar fuzzy weak bi- ideal of a gamma near algebra *N* over a tripolar fuzzy field $F=(\alpha_F,\beta_F,\gamma_F)$ in X then $f^{-1}(T)=(f^{-1}(\alpha_T),f^{-1}(\beta_T),f^{-1}(\gamma_T))$ is a tripolar fuzzy weak bi- ideal of a gamma near algebra M.

Proof: Suppose $T=(\alpha_T,\beta_T,\gamma_T)$ is a tripolar fuzzy weak bi-ideal of N and $p,q\in M,\eta,\nu\in\Gamma$ then

$$f^{-1}(\alpha_T(p+q))=\alpha_T(f(p+q))\geq\min(\alpha_T(f(p),f(q))=\min(f^{-1}(\alpha_T(p)),f^{-1}(\alpha_T(q))).$$

$$f^{-1}(\alpha_T(p\eta q\nu r)=\alpha_T(f(p\eta q\nu r))\geq\min(\alpha_T(f(p\eta q\nu r))\geq\min(f^{-1}(\alpha_T(p\eta q\nu r))$$
$$=\min(f^{-1}(\alpha_T(p),f^{-1}(\alpha_T(q)),f^{-1}(\alpha_T(r))).$$

$$f^{-1}(\alpha_T(\lambda p))=\alpha_T(f(\lambda p))\geq\min(\alpha_T(f(\lambda),f(p))=\min(f^{-1}(\alpha_F(\lambda),f^{-1}(\alpha_T(p))).$$

$$f^{-1}(\alpha_F(1))=\alpha_F(f(1))\geq\min(\alpha_F(f(1)))=\alpha_T(f(p)).$$

$$f^{-1}(\beta_T(p\eta q\nu r))=\beta_T(f(p\eta q\nu r))\leq\max(\beta_T(f(p\eta q\nu r))\leq\max(f^{-1}(\beta_T(p\eta q\nu r))$$
$$=\max(f^{-1}(\beta_T(p)),f^{-1}(\beta_T(q)),f^{-1}(\beta_T(r))).$$

$$f^{-1}(\beta_T(\lambda p))=\beta_T(f(\lambda p))\leq\max(\beta_T(f(\lambda p)))\leq\max(\beta_T(f(\lambda),f(p)))=\max(f^{-1}(\beta_F(\lambda),f^{-1}(\beta_T(p))).$$

$$f^{-1}(\beta_F(1))=\beta_F(f(1))\leq\max(\beta_F(f(1))\leq\max(\beta_T(p))=\beta_T(p).$$

$$f^{-1}(\gamma_T(p+q))=\gamma_T(f(p+q))\leq\max(\gamma_T(f(p),f(q)))=\max(f^{-1}(\gamma_T(p),f^{-1}(\gamma_T(q)).$$

$$f^{-1}(\gamma_T((p\eta q\nu r))=\gamma_T(f(p\eta q\nu r))\leq\max(\gamma_T(f(p\eta q\nu r))\leq\max(\gamma_T(f(p),f(q),f(r)))$$
$$=\max(f^{-1}(\gamma_T(p)),f^{-1}(\gamma_T(q)),f^{-1}(\gamma_T(r))).$$

$$f^{-1}(\gamma_T(\lambda p))=\gamma_T(f(\lambda p))\leq\max(\gamma_T(f(\lambda p)))\leq\max(\gamma_T(f(\lambda),f(p)))=\max(f^{-1}(\gamma_F(\lambda)),f^{-1}(\gamma_T(p))).$$

$$f^{-1}(\gamma_F(1))=\gamma_F(f(1))\leq\max(\gamma_F(f(1))\leq\max(\gamma_T(p))=\gamma_T(p).$$

Thus $f^{-1}(B)=(f^{-1}(\alpha_T),f^{-1}(\beta_T),f^{-1}(\gamma_T))$ is a tripolar fuzzy weak bi-ideal of M.

Theorem 3.8: The cartesian product of two tripolar fuzzy weak bi- ideals of a gamma near algebras over a tripolar fuzzy field $F=(\alpha_F,\beta_F,\gamma_F)$ in X is also a tripolar fuzzy weak bi-ideal.

Proof: Let A and B be two tripolar fuzzy weak bi- ideals of a gamma near algebras Y, Z.

Then their cartesian product $A\times B$ is also a tripolar fuzzy weak bi-ideal of a gamma near algebra $Y\times Z$. Let $(p_1,p_2),(q_1,q_2)\in Y\times Z$.

$$\alpha_{A\times B}(p_1,p_2)+(q_1,q_2)\geq\min(\alpha_A(p_1+q_1),\alpha_B(p_2+q_2))\geq\min(\min(\alpha_A(p_1),\alpha_A(q_1)),\min(\alpha_B(p_2),\alpha_B(q_2))$$
$$\geq\min(\min(\alpha_A(p_1),\alpha_B(p_2)),\min(\alpha_A(q_1),\alpha_B(q_2)))=\min(\alpha_{A\times B}(p_1,p_2),\alpha_{A\times B}((q_1,q_2)).$$

$$\alpha_{A\times B}(p_1,p_2)\eta(q_1,q_2)\nu(r_1,r_2) \geq \min(\alpha_{A\times B}((p_1,p_2)\eta((q_1,q_2)\nu(r_1,r_2))) \geq \min(\alpha_{A\times B}(p_1\eta q_1\nu r_1, p_2\eta q_2\nu r_2))$$
$$\geq \min(\alpha_A(p_1\eta q_1\nu r_1), \alpha_B(p_2\eta q_2\nu r_2))$$
$$\geq \min(\min(\alpha_A(p_1),\alpha_A(q_1),\alpha_A(r_1)), \min(\alpha_B(p_2),\alpha_B(q_2),\alpha_B(r_2))$$
$$\geq \min(\min(\alpha_A(p_1),\alpha_B(p_2)), \min(\alpha_A(q_1),\alpha_B(q_2)), \min(\alpha_A(r_1),\alpha_B(r_2)))$$
$$= \min(\alpha_{A\times B}(p_1,p_2), \alpha_{A\times B}(q_1,q_2), \alpha_{A\times B}(r_1,r_2)).$$

$$\alpha_{A\times B}(\lambda p_1,\lambda p_2) \geq \min(\alpha_{A\times B}(\lambda p_1,\lambda p_2)) \geq \min(\min(\alpha_F(\lambda),\alpha_A(p_1,),\alpha_B(p_2))$$
$$\geq \min(\alpha_F(\lambda), \min(\alpha_A(p_1),\alpha_B(p_2)) = \min(\alpha_F(\lambda), \alpha_{A\times B}(p_1,p_2)).$$

$$\beta_{A\times B}(p_1,p_2)+(q_1,q_2) \leq \max(\beta_A(p_1+q_1),\beta_B(p_2+q_2)) \leq \max(\max(\beta_A(p_1),\beta_A(q_1)), \max(\beta_B(p_2),\beta_B(q_2))$$
$$\leq \max(\max(\beta_A(p_1),\beta_B(p_2)), \max(\beta_A(q_1),\beta_B(q_2))) = \max(\beta_{A\times B}(p_1,p_2), \beta_{A\times B}((q_1,q_2)).$$

$$\beta_{A\times B}(p_1,p_2)\eta(q_1,q_2)\nu(r_1,r_2) \leq \max(\beta_{A\times B}(p_1,p_2)\eta(q_1,q_2)\nu(r_1,r_2)) \leq \max(\beta_{A\times B}(p_1\eta q_1\nu r_1, p_2\eta q_2\nu r_2))$$
$$\leq \max(\beta_A(p_1\eta q_1\nu r_1), \beta_B(p_2\eta q_2\nu r_2))$$
$$\leq \max(\max(\beta_A(p_1),\beta_A(q_1),\beta_A(r_1)), \max(\beta_B(p_2),\beta_B(q_2),\beta_B(r_2)))$$
$$\leq \max(\beta_A(p_1),\beta_B(p_2),\beta_A(q_1),\beta_B(q_2),\beta_A(r_1),\beta_B(r_2))$$
$$= \max(\beta_{A\times B}(p_1,p_2), \beta_{A\times B}(q_1,q_2), \beta_{A\times B}(r_1,r_2)).$$

$$\beta_{A\times B}(\lambda p_1,\lambda p_2) = (\beta_{A\times B}(\lambda p_1,\lambda p_2)) \leq \max(\max(\beta_F(\lambda),\beta_A(p_1),\beta_B(p_2))$$
$$\leq \max(\beta_F(\lambda), \max(\beta_A(p_1),\beta_B(p_2)) = \max(\beta_F(\lambda), \beta_{A\times B}(p_1,p_2)).$$

$$\gamma_{A\times B}(p_1,p_2)+(q_1,q_2) \leq \max(\gamma_A(p_1+q_1),\gamma_B(p_2+q_2)) \leq \max(\max(\gamma_A(p_1),\gamma_A(q_1)), \max(\gamma_B(p_2),\gamma_B(q_2))$$
$$\leq \max(\max(\gamma_A(p_1),\gamma_B(p_2)), \max(\gamma_A(q_1),\gamma_B(q_2))) = \max(\gamma_{A\times B}(p_1,p_2), \gamma_{A\times B}((q_1,q_2)).$$

$$\gamma_{A\times B}(p_1,p_2)\eta(q_1,q_2)\nu(r_1,r_2) \leq \max(\gamma_{A\times B}(p_1,p_2)\eta(q_1,q_2)\nu(r_1,r_2)) \leq \max(\gamma_{A\times B}(p_1\eta q_1\nu r_1, p_2\eta q_2\nu r_2))$$
$$\leq \max(\gamma_A(p_1\eta q_1\nu r_1), \gamma_B(p_2\eta q_2\nu r_2))$$
$$\leq \max(\max(\gamma_A(p_1),\gamma_A(q_1),\gamma_A(r_1)), \max(\gamma_B(p_2),\gamma_B(q_2),\gamma_B(r_2))$$
$$\leq \max(\max(\gamma_A(p_1),\gamma_B(p_2)), \max(\gamma_A(q_1),\gamma_B(q_2)), \max(\gamma_A(r_1),\gamma_B(r_2)))$$
$$= \max(\gamma_{A\times B}(p_1,p_2), \gamma_{A\times B}(q_1,q_2), \gamma_{A\times B}(r_1,r_2)).$$

$$\gamma_{A\times B}(\lambda p_1,\lambda p_2) \leq \max(\gamma_{A\times B}(\lambda p_1,\lambda p_2)) \leq \max(\gamma_A(\lambda p_1),\gamma_B(\lambda p_2)) \leq \max(\gamma_F(\lambda),\gamma_A(p_1),\gamma_B(p_2))$$
$$\leq \max(\max(\gamma_F(\lambda),\gamma_A(p_1), \max(\gamma_F(\lambda),\gamma_B(p_2))$$
$$\leq \max((\gamma_F(\lambda), \max(\gamma_A(p_1),\gamma_B(p_2)) = \max(\gamma_F(\lambda), \gamma_{A\times B}(p_1,p_2)).$$

By the definition $\alpha_F(1) \geq \alpha_T(x), \beta_F(1) \leq \beta_T(x), \gamma_F(1) \leq \gamma_T(x)$ for every $x \in Y$, and $\alpha_F(1) \geq \alpha_B(x'), \beta_F(1) \leq \beta_B(x'), \gamma_F(1) \leq \gamma_B(x')$ for every $x' \in Z$. Then

$$\alpha_F(1) \geq (\alpha_A(x), \alpha_B(x') = \alpha_{A\times B}(x,x');$$
$$\beta_F(1) \leq (\beta_A(x), \beta_B(x') = \beta_{A\times B}(x,x');$$
$$\gamma_F(1) \leq (\gamma_A(x), \gamma_B(x') = \gamma_{A\times B}(x,x');$$

Hence the cartesian product of two tripolar fuzzy weak bi-ideals of a tripolar fuzzy near algebras is also a tripolar fuzzy weak bi-ideal.

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