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# DRAG ON AN IMPERMEABLE CYLINDER EMBEDDED IN A SPARSELY PACKED POROUS MEDIUM IN PRESENCE OF MAGNETIC FIELD

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**ABSTRACT**

The present devoted to find an expression for drag power qualified by the solid cylinder placed in a sparsely packed porous region with additional magnetic field in the transverse direction to the flow. The flow is governed by the Brinkman model (To take care of the boundary effects that exists on the boundaries, the Brinkman modified the Darcy equation by considering a Laplacian term in velocity to Darcy law) and solved analytically using a uniform shear far from the cylinder. It is found that, amplifying in the permeability of a porous region is to decline the drag on the solid cylinder. Further, a reversed behavior is noticed in the hydrodynamic force when the magnetic field is raised and the same is represented graphically.

***KEYWORDS:***

Brinkman model;
Porous region;
Hydrodynamic force;
Uniform shear.



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## 1. INTRODUCTION

The viscous fluid flow past impermeable objects placed in porous media needs a broad perspective of how the flow influences the hydrodynamic drag force experienced by the objects. The detailed information of these drag forces can help in improving and developing the existing models. The precise computation of drag consequences in cost effective design of automobiles, chimneys, towers, buildings, hydraulic structures etc. Further, the problem of acquifier, oil technology, geo-physical, biomechanical and engineering ete and Impact of nonlinear thermal radiation and gyrotactic microorganisms on the Magneto-Burgers nanofluid, Behavior of stratifications and convective phenomena in mixed convection flow of 3D Carreaunanofluid with radiative heat flux, A numerical approach, Simultaneous impact of nonlinear radiative heat flux and Arrhenius activation energy in flow of chemically reacting Carreaunanofluid.

In the literature, studies are found on the fluid flow past a cylinder in porous media using Darcy's or Brinkman equation under different boundary conditions. [1]-[3] they have used Darcy's equation to describe the flow field.The problem of Stokes flow past porous bodies have been studied [4]-[9] using Brinkman model in porous region to describe the motion [10]-[11] proposed a cell model in which two concentric cylinders serve as the model for fluid moving through an assemblage of circular cylinders. The Kuwabara model assumes uniform velocity condition and vanishing of vorticity at the cell surface. [12] have considered the problem of an incompressible steady viscous flow past a circular cylinder embedded in a constant-porosity medium based on the Brinkman model, and obtained a closed- form exact solution of the stream function of the Brinkman equation. [13] Has studied on mixed convection about a horizontal cylinder in a fluid-saturated porous medium using Darcy model. The drag on flow past a cylinder with slip was evaluated [14]. Stokes flow past a swarm of porous circular cylinders with Happel and kuwabara boundary conditions was discussed [15]. [16] Studied problem of the Stokes flow past a swarm of porous nano cylindrical particles enclosing a solid cylindrical core with Kuwabara boundary condition. Recently [17] studied flow of conducting fluid on solid core surrounded by porous cylindrical region in the presence of transverse magnetic field. [18] Studied the drag coefficient of a cylinder with varying flow velocities and diameters. They have used direct weighing method and pressure distribution analysis around the cylinder to calculate the drag on the cylinder and also found that weighing method is more accurate than the pressure distribution around the cylinder.

This paper presents the analytical study of fluid flow past an impermeable cylinder embedded in a sparsely packed porous medium in presence of transverse magnetic field. The Brinkman equation is considered to describe the flow in porous region, specifying a uniform shear away from the impermeable cylinder. The drag force experienced by an impermeable cylinder embedded in a porous medium is evaluated. The dependence of drag coefficient on non - dimensional parameters is discussed and presented graphically.

## 2. RESEARCH METHOD

Let as a steady, incompressible fluid flow past an impermeable cylinder of radius 'a' placed in a sparsely packed porous medium in the presence of a transverse magnetic field.The schematic representation is show in Figure 1. Under the assumptions and approximations made together with small electrical conductivity, the characteristic of the conducting fluid run in porous region is given by the continuity equation, equation of modified Brinkman as:

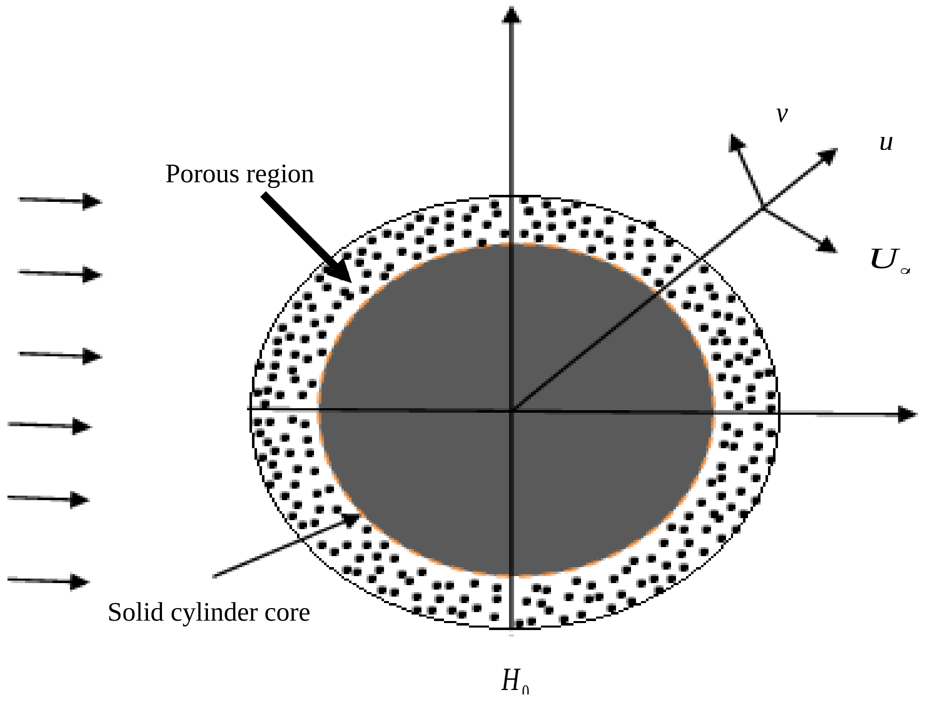


Figure1. Physical represence of the problem

$$\nabla \cdot \vec{q} = 0. \tag{1}$$

$$\nabla p = -\frac{\mu}{k}\vec{q} + \mu_h^2\ \sigma_e\left(\vec{q} \times \vec{H}\right) \times \vec{H} + \bar{\mu}\ \nabla^2 \vec{q}. \tag{2}$$

where $\vec{q} = (u, v)$ is the rate in the porous region, $\vec{H}$ is the uniform magnetic field, $\mu$ is the viscosity of the fluid, $\bar{\mu}$ is the Brinkman viscosity $\mu_h$ is the magnetic permeability, $\sigma_e$ -electrical conductivity, $k$ - permeability of the porous region, $p$ is the hydrostatic pressure of the porous region Here we assumed that $\bar{\mu} = \mu$. And regular magnetic field is $H_0$.

The governing equations of the problem described by equations (1) and (2) can be representing in conditions of non-dimensional parameters.

$$\vec{q}^* = \frac{\vec{q}}{u_\infty}, \quad p^* = \frac{ap}{\bar{\mu} u_\infty}, \quad \vec{H}^* = \frac{\vec{H}}{H_0} \tag{3}$$

Using the transformation from equation (3), the equations (1) and (2) are non-dimensionalised for the porous region, and the corresponding equations in cylindrical polar co-ordinate system as:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial \theta}(v) = 0 \tag{4}$$

$$-\frac{\partial p}{\partial r} = c^2 u - \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right]. \tag{5}$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta}=c^{2}v-\left[\frac{\partial^{2}v}{\partial r^{2}}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{1}{r^{2}}\frac{\partial^{2}v}{\partial \theta^{2}}+\frac{2}{r^{2}}\frac{\partial u}{\partial \theta}-\frac{v}{r^{2}}\right]. \qquad (6)$$

Where $c^2=\sigma^2+H^2$, in which $\sigma=\frac{a}{\sqrt{k}}$ is the porous parameter. $u, v$ are the velocity components in the radial and transverse directions respectively. Since flow is axi-symmetric, the stream function $\psi(r,\theta)$ is introduced, such that the equation of continuity is satisfied in cylindrical polar co-ordinate system anddefined as:

$$u=-\frac{1}{r}\frac{\partial \psi}{\partial \theta};\quad v=\frac{\partial \psi}{\partial r}. \qquad (7)$$

By eliminating the pressure term from equations (5) and (6) of porous region then by cross differentiation, a fourth order partial differential equation is obtained in terms of stream function as:

$$\nabla^{4}\psi-c^{2}\nabla^{2}\psi=0. \qquad (8)$$

Where $\nabla^{2}=\frac{\partial^{2}}{\partial r^{2}}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^{2}}\frac{\partial^{2}}{\partial \theta^{2}}$ is the Laplacian operator in cylindrical co-ordinate system and

$$c^{2}=\left(\frac{a^{2}}{k}+\frac{\mu_h^{2}\sigma_e H_0{}^{2}a^{2}}{\overline{\mu}}\right) \qquad (9)$$

**3. BOUNDARY CONDITIONS**

To solve the above governing equation we considered the boundary equations: with no-slip on solid surface of the cylinder is given by

$$\frac{\partial}{\partial \theta}\psi(r,\theta)=\frac{\partial}{\partial r}\psi(r,\theta)=0 \qquad at\ r=1. \qquad (10)$$

Far away from the cylinder the flow has uniform velocity

$$\psi(r,\theta)\sim r\sin\theta, \text{ as } r\to\infty. \qquad (11)$$

**4. SYSTEM OF SOLUTION**

The boundary condition from equation (11) suggests the following similarity solution

$$\psi(r,\theta)=f(r)\sin\theta. \qquad (12)$$

Substituting equation (12) in (8) the fourth order partial differential equation in terms of stream functions $\psi(r,\theta)$ reduces to fourth order ordinary differential equation with variable co efficient in $f(r)$ are follows:

$$f^{iv}(r)+\frac{2}{r}f'''(r)-\frac{3}{r^{2}}f''+\frac{3}{r^{3}}f'(r)-\frac{3}{r^{4}}f(r)-c^{2}\left(f''(r)+\frac{1}{r}f'-\frac{1}{r^{2}}f(r)\right)=0. \qquad (13)$$

The corresponding boundary conditions in terms of $f(r)$ from equations (10) and (11) reduces to No-slip condition at the surface of the porous cylinder is given by

$$f(1)=0, \qquad f'(1)=0. \qquad (14)$$

Further, the uniform velocity far away from the boundary, from equation (12) reduces to:

$f(r) \sim r$ as $r \to \infty$ . (15)

Where the prime denotes differentiation with respect to r

We denote $g(r) = f''(r) + \frac{1}{r} f'(r) - \frac{1}{r^2} f(r)$ (16)

Substitution of equation (16) in equation (13), it reduces to second order ordinary differential equation in $g(r)$ as,

$$g''(r) + \frac{1}{r} g'(r) - \left(c^2 + \frac{1}{r^2}\right) g(r) = 0 . \quad (17)$$

The solution for the equation (13) is obtained as

$$g(r) = BK_1(cr) + CI_1(cr) . \quad (18)$$

Where, $K_1(cr)$ and $I_1(cr)$ are the modified Bessel functions of second and first kind of order respectively and B and C are arbitrary constants.From equation (11) on equation (18) we get C=0 and using (16) becomes

$$BK_1(cr) = f''(r) + \frac{1}{r} f'(r) - \frac{1}{r^2} f(r) . \quad (19)$$

Equation (19) is a linear differential equation with variable co-efficient; its general solution can be obtained by the method of variation of parameters and is given by:

$$\frac{A}{r} + Br + CK_1(rc) = f(r) \quad (20)$$

Where A, B and C are arbitrary constants to be determined using the boundary conditions (14) and (15) and hence obtained solution is:

$$f(r) = r - \left(1 + \frac{2K_1(c)}{cK_0(c)}\right)\frac{1}{r} + \frac{2}{cK_0(c)} K_1(rc) \cdot \quad (21)$$

Therefore equation (12) reduces to:

$$\psi(r,\theta) = \left( r - \left(1 + \frac{2K_1(c)}{cK_0(c)}\right)\frac{1}{r} + \frac{2}{cK_0(c)} K_1(rc) \right) \sin\theta . \quad (22)$$

Equation (22) shows that function for the present problem based on the Brinkman model is a function of porous parameter and Hartmann number

The shearing stress at any point on the surface of the cylinder is given by

$$\tau_{r\theta} = \overline{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) \quad (23)$$

On dimensionless, equation (23) reduces to

$$\frac{\tau_{r\theta}}{\frac{\overline{\mu} U_\infty}{a}} = -\left(\frac{1}{r}\frac{\partial u}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) \quad (24)$$

On the surface of the cylinder (i.e r =1), the shearing stress becomes

$$\frac{\tau_{r\theta}}{\frac{\bar{\mu}U_{\infty}}{a}} = -r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) \tag{25}$$

$$\tau_{r\theta} = \frac{\bar{\mu}U_{\infty}}{a}(1+S)\sin\theta. \tag{26}$$

Where $S = \frac{16c+6}{8c^2-c-3} - \frac{c}{2}\left[\frac{16c+22}{8c-1}\right]$

**5. EVALUATION OF THE DRAG FORCE**

The drag force F experienced by a impermeable cylinder of radius 'a' embedded in porous medium can be evaluated by integrating the stresses over the impermeable cylinder:

$$F_t = \int_0^{2\pi}\int_0^{\pi}\{(\tau_{r\theta})_{r=a}\sin\theta\}R^2\, d\theta\, d\phi \tag{27}$$

Further, equation (25) reduces to as:

$$F_t = 6\pi\bar{\mu}U_{\infty}a(1+S). \tag{28}$$

Also, the drag coefficient can be defined as:

$$C_D = \frac{-F_t}{\frac{1}{2}\rho U_{\infty}^2 a^2 \pi} \tag{29}$$

Substitution of equation (28) in equations (29) reduces to:

$$C_D = -\frac{24}{R_e}(1+S)\ and\ R_e = \frac{2\rho U_{\infty}a}{\bar{\mu}} \tag{30}$$

Where $R_e$ is the Reynolds number and if S=0, then equation (28) reduces to:

$$F_t = 6\pi\bar{\mu}U_{\infty}a \tag{31}$$

This result for the drag reported earlier by Stokes [19]

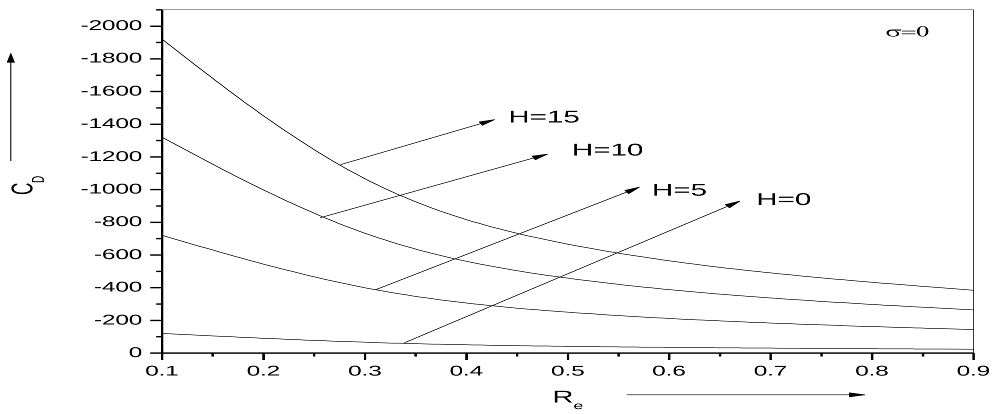


**Figure2**. Variation of the Drag coefficient with Hartmann number H for fixed porous parameter σ=0.

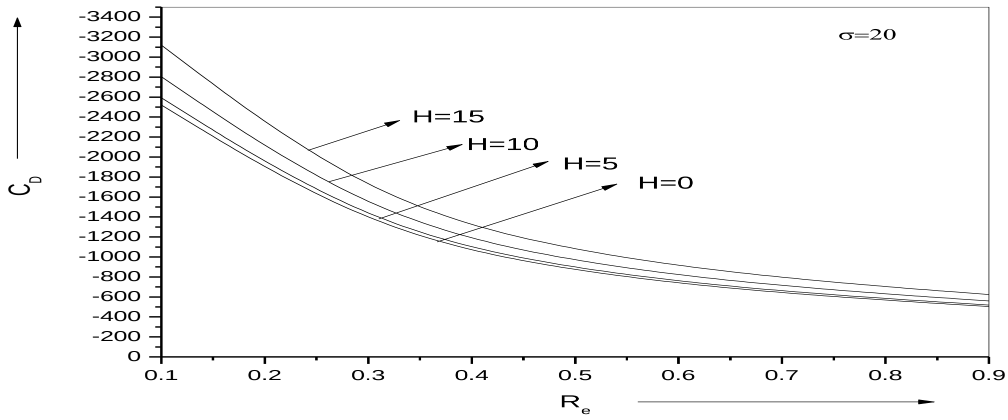


**Figure 3.** Variation of the Drag coefficient with Hartmann number H for fixed porous parameter σ=20.

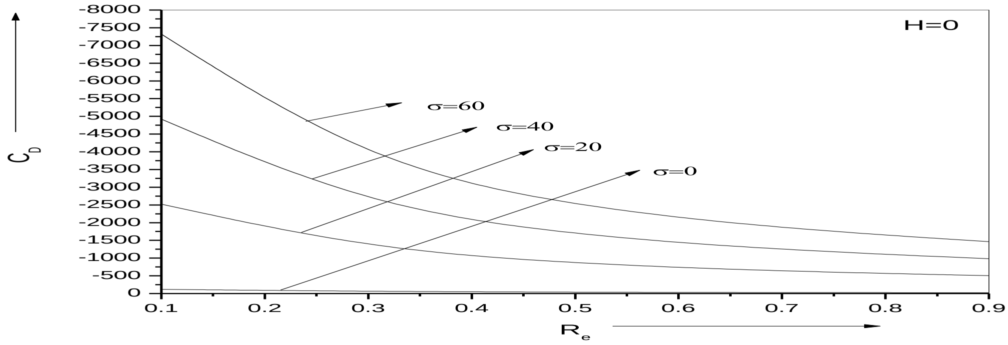


**Figure 4.** Variation of the Drag coefficient with porous parameter σ for fixed Hartmann number H=0.

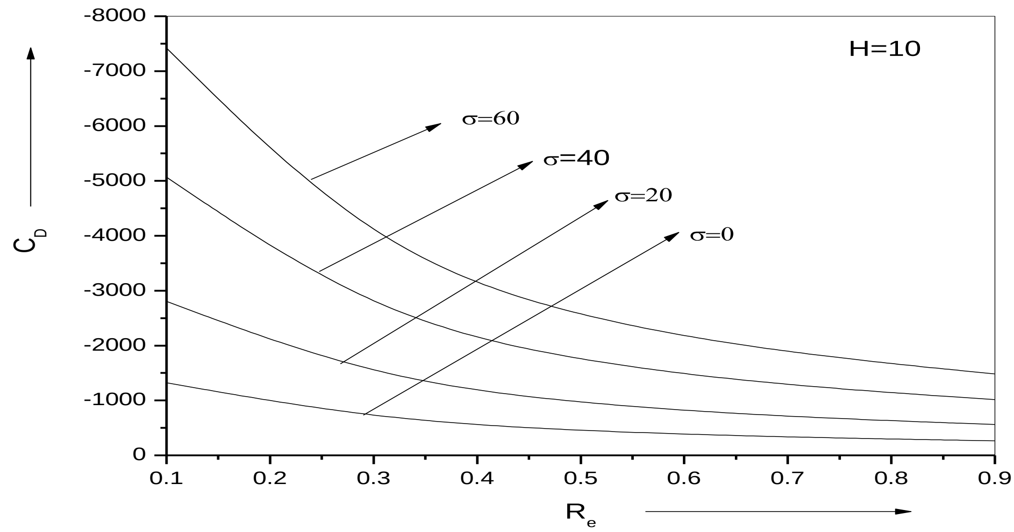


**Figure 5**. Variation of the Drag coefficient with porous parameter σ for fixed Hartmann number H=10.

**6. RESULTS AND ANALYSIS**

Here we study the steady run of viscous, incompressible fluid in attendance of transverse magnetic field past an impermeable cylinder placed in a sparsely packed cylindrical porous region. The drag qualified by a cylinder embedded in porous medium has been resulting. The difference of drag coefficient for different values of porous parameter and Hartmann number is analyzed. Also, the expression for the drag co-efficient $C_D$ is obtained in absence of magnetic field as a limiting case.

From figure 2 and 3, we noticed that the drag co-efficient $C_D$ decreases with increase in Hartmann number H for fixed porous parameter σ =0, σ =20 near the solid surface and maintains asymptotic behavior away from the surface. Further, figure 4 and 5 shows that the increase in porous parameter σ, drag co-efficient $C_D$ decreases for fixed Hartmann number H =0, 10.

**7. CONCLUSION**

In this paper we study the variation of drag coefficient with variation of porous parameter and Hartmann number for the steady flow of viscous, incompressible fluid in presence of magnetic field past an impermeable cylinder placed in a sparsely packed cylindrical porous region. Hence it is found that the increase in porous parameter and Hartmann number with drag coefficient decreases.

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