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BIFURCATION AND CHAOS CONTROL BY OGY METHOD IN BURGERS MAPPING

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ABSTRACT

In this paper, the bifurcation examination for Burgers mapping has been considered using Schur-Cohn criterion. The existence and stability of the fixed points of this map is derived. The bifurcations of periodic points and the states of presence for pitchfork bifurcation, flip bifurcations are determined by utilizing Schur-Cohn criterion. The numerical reenactments not just demonstrate the consistence with the hypothetical examination yet in addition show the complex dynamical practices. Additionally chaos is controlled utilizing OGY method.

KEYWORDS:

Schur-Cohn criterion;
pitchfork bifurcation;
flip bifurcation;
OGY method;

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1. INTRODUCTION:

The mapping described here arose out of an early attempt by Burgers to illustrate the transition from laminar
to turbulent flow by means of a pair of first order non-linear differential equations the fixed points of which change character at a critical value of a parameter. It is a discretisation of a

pair of coupled differential equations which were utilized by Burgers to represent the importance of the idea of bifurcation to the investigation of hydrodynamics streams[1],[2],[3],[4],[5],[6],[10],[11][17] Burgers' equations are

$$\frac{dx}{dt} = P - \nu x - y^2$$
$$\frac{dy}{dt} = xy - \nu y$$

and the discretised equations, with a minor change of parametrisation, are

$$x_{i+1} = (1 - \varepsilon\nu)x_i - \varepsilon y_i{}^2$$
$$y_{i+1} = (1 + \varepsilon\mu + \varepsilon x_i)y_i$$

By a suitable scaling transformation, $x \to \varepsilon x, y \to \varepsilon y$, the step size parameter ε can be eliminated and equations are replaced by

$$x_{i+1} = (1 - \nu)x_i - y_i{}^2$$
$$y_{i+1} = (1 + \mu + x_i)y_i$$

The Burgers mapping is defined in the following way:

$$x_{n+1} = (1 - \nu)x_n - y_n{}^2$$
$$y_{n+1} = (1 + \mu)y_n + x_n y_n$$

(1.1)

2. STABILITY ANALYSIS:

Let $\bar{x}$ be a steady state of a system for a given parametric value $\bar{u}$.

(a) If all the eigenvalues λ_i of the $n \times n$ Jacobian matrix $\bar{J} = J(\bar{u}, \bar{x})$ of the considered system lie in the open unit disk, i.e. $|\lambda_i| < 1$ for all i, then $\bar{x}$ is asymptotically stable.

(b) If the matrix $\bar{J}$ has at least one eigenvalue λ_0 outside the open unit disk, i.e. $|\lambda_0| > 1$, then $\bar{x}$ is unstable. [2],[18],[19],[20]

Let $A = \lambda^n + a_{n-1}\lambda^{n-1} + ... + a_0$ be the characteristic polynomial of $\bar{J}$. The eigenvalues of $\bar{J}$ are simply the roots of the polynomial A for **λ**. So the problem of stability analysis can be reduced to that of determining whether all the roots of A lie in the open unit disk $|\lambda| < 1$.

2.1 Schur-Cohn criterion:

The Schur-Chon criterion provides an alternative, more direct test for the stability of discrete dynamical systems.This criterion is expressed in terms of certain determinants formed from the coefficients of A. Consider the sequence of determinants

$D_1^{\pm}, D_2^{\pm}, \ldots, D_n^{\pm}$, where

$$D_i^{\pm} = \left| \begin{pmatrix} 1 & a_{n-1} & a_{n-2} & \cdots & a_{n-i+1} \\ 0 & 1 & a_{n-1} & \cdots & a_{n-i+2} \\ 0 & 0 & 1 & \cdots & a_{n-i+3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \pm \begin{pmatrix} a_{i-1} & a_{i-2} & \cdots & a_1 & a_0 \\ a_{i-2} & a_{i-3} & \cdots & a_0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1 & a_0 & \cdots & 0 & 0 \\ a_0 & 0 & \cdots & 0 & 0 \end{pmatrix} \right|$$

2.2 Theorem:

The characteristic polynomial A has all its roots inside the unit open disk if and only if

(a) $A(1) > 0$ and $(-1)^n A(-1) > 0$,

(b) $D_1^{\pm} > 0, D_3^{\pm} > 0, \ldots, D_{n-3}^{\pm} > 0, D_{n-1}^{\pm} > 0,$ (when n is even), or

$D_2^{\pm} > 0, D_4^{\pm} > 0, \ldots, D_{n-3}^{\pm} > 0, D_{n-1}^{\pm} > 0$ (when n is odd).

In this subsection, we recall some important bifurcations and explain how to reduce the problems of bifurcation analysis to algebraic problems. [2],[18],[19],[20]

2.3 Period doubling bifurcation:

At this bifurcation, the system switches to a new behaviour with twice the period of the original system. A series of period doubling bifurcations may lead the system from order to chaos. In this situation, the Jacobian matrix $\bar{J}$ has one real eigenvalue which equals -1, and the other eigenvalues of $\bar{J}$ are all inside the unit cicle. [2],[18],[19],[20]

Obviously , a necessary and sufficient condition for the characteristic polynomial A of $\bar{J}$ to have one real root -1 and all other roots inside the unit circle is

(a) $A(1) > 0$ and $A(-1) = 0$,

(b) $D_1^{\pm} > 0, D_3^{\pm} > 0, \ldots, D_{n-3}^{\pm} > 0, D_{n-1}^{\pm} > 0,$ (when n is even), or

$D_2^{\pm} > 0, D_4^{\pm} > 0, \ldots, D_{n-3}^{\pm} > 0, D_{n-1}^{\pm} > 0$ (when n is odd).

2.4 Stationary bifurcation:

If the Jacobian matrix $\bar{J}$ of system has one real eigenvalue equal to 1, then the system may undergo a saddle-node (also called fold bifurcation), transcritical, or pitchfork bifurcation. These bifurcations are all called stationary bifurcations. Pitchfork bifurcation occurs for dynamical systems with symmetry; in this case, the number of steady state changes from one to three, or from three to one, while the number of stable steady states from one to two, or from one to zero. [2],[18],[19],[20]

Replacing condition (a) above by $A(1)=0$ and $(-1)^n A(-1)>0$, one can obtain the condition under which system may undergo a stationary bifurcation, but the determination of the type of stationary bifurcation for a concrete system requires further analysis.

3. STABILITY OF FIXED POINTS AND BIFURCATIONS:

We initially decide the presence of the fixed points of the map (1.1), then examine their stability by figuring the eigenvalues for the Jacobian matrix of the map at each fixed point and sufficient conditions of existence for pitchfork bifurcation and flip bifurcation by utilizing bifurcation hypothesis.[3],[4],[5],[8],[9],[10],[11]

Jacobian of the map is given by $$J=\begin{pmatrix}1-\nu & -2y\\ y & 1+\mu+x\end{pmatrix}$$

(3.1)

$|J|=1-\nu+\mu-\nu\mu+x-\nu x+2y^2$

The Burgers map has three fixed points say $(x_0,y_0),(x_1,y_1)$ and (x_2,y_2) whose co-ordinates are given by the solutions of $[(1-\nu)x-y^2,(1+\mu)y+xy]=(x,y)$ and they are found to be $(0,0),(-\mu,\sqrt{\nu\mu})$ and $(-\mu,-\sqrt{\nu\mu})$

The fixed points exists for either $\nu>0,\mu>0$ or $\nu<0,\mu<0$.

The stability theory intimately connected with the Jacobian matrix of the map, and that the trace of the Jacobian matrix is the sum of its eigenvalues and the product of the eigenvalues equals the Jacobian determinant. [17],[18],[19],[20]

3.1 Pitchfork bifurcation at (0,0)

Here, we consider the Jacobian matrix at $(0,0)$ which has the form

$$J=\begin{pmatrix}1-\nu & 0\\ 0 & 1+\mu\end{pmatrix}.$$

The characteristic polynomial of J is $A(\lambda)=\lambda^2+\lambda(\nu-\mu-2)+(1+\mu-\nu-\nu\mu)$

(3.2)

By Schur-Cohn criterion, for the map, stationary bifurcation occurs if $A(1)=0$ and $(-1)^n A(-1)>0$.

Here, $A(1)=-\nu\mu \Rightarrow \mu=0$ if $\nu\neq 0$.

and $(-1)^n A(-1) = (-1)^2(4 + 2\mu - 2\nu - \nu\mu) > 0 \Rightarrow 4 + 2\mu - 2\nu - \nu\mu > 0$.

Again, the jacobian matrix at (0, 0) has eigenvalues $\lambda_1 = 1 - \nu$ and $\lambda_2 = 1 + \mu$.

Criterion: Pitchfork bifurcation occurs when one of the eigenvalue becomes 1. [17],[18],[19],[20]

Considering that μ is a bifurcation parameter and (0, 0) is a non-hyperbolic fixed point for $\mu = 0$ or $\mu = -2$

If $\mu = 0$ then the Jacobian matrix at (0, 0) has two eigenvalues $\lambda_1 = 1 - \nu$ and $\lambda_2 = 1$.

If $0 < \nu < 2$ and $-2 < \mu < 0$, then (0,0) is stable fixed point. And for $\nu > 2$ and $\mu > 0$, (0, 0) is an unstable fixed point and we get two stable fixed points $(-\mu, \sqrt{\nu\mu})$ and $(-\mu, -\sqrt{\nu\mu})$.

when $\mu = 0$ and $\nu \neq 0$, the map (1.1) undergoes a pitchfork bifurcation at (0, 0). And the map has only one fixed point. [3],[4],[5],[10],[11]

3.2 Pitchfork Bifurcation at fixed point $(\mu, \sqrt{\nu\mu})$

The Jacobian matrix at $(-\mu, \sqrt{\nu\mu})$ has the form, $J = \begin{pmatrix} 1-\nu & -2\sqrt{\nu\mu} \\ \sqrt{\nu\mu} & 1 \end{pmatrix}$.

The characteristic polynomial of J is $A(\lambda) = \lambda^2 + \lambda(\nu - 2) + (2\nu\mu - \nu + 1)$.

By Schur-Cohn criterion [3],[13],[16],[18], for the map, stationary bifurcation occurs if $A(1) = 0$ and $(-1)^n A(-1) > 0$. Here, $A(1) = 2\nu\mu = 0 \Rightarrow \mu = 0$ if $\nu \neq 0$;

and $(-1)^n A(-1) = (-1)^2(4 - 2\nu + 2\nu\mu) > 0 \Rightarrow 4 - 2\nu + 2\nu\mu > 0 \Rightarrow \mu > \dfrac{\nu - 2}{\nu}$.

Again, the jacobian matrix at (0, 0) has eigenvalues $\lambda_1 = 1 - \nu$ and $\lambda_2 = 1 + \mu$.

If λ_1, λ_2 are the eigenvalues of J then we have $\lambda_1 + \lambda_2 = 2 - \nu + \mu + x$

(3.3)

$$\lambda_1\lambda_2 = 1 - \nu + \mu - \nu\mu + x - \nu x + 2y^2$$

(3.4)

Now,for pitchfork bifurcation [8],[9] to take place, one of the eigenvalues must be 1.

So if we take $\lambda_2 = 1$ equation (3.3) and (3.4) implies

$$\lambda_1 = 1 - \nu + \mu + x \qquad (3.5)$$

$$\lambda_1 = 1 - \nu + \mu - \nu\mu + x - \nu x + 2y^2 \qquad (3.6)$$

$$(3.5)\ and\ (3.6) \Rightarrow 1 - \nu + \mu + x = 1 - \nu + \mu - \nu\mu + x - \nu x + 2y^2$$

(3.7)

Now, putting $x = -\mu, y = \sqrt{\nu\mu}$ in equation (3.7) and solving for b, we get $\mu = 0, where\ \nu \neq 0$

Also the eigenvalues of the Jacobian matrix at the point $(-\mu,\sqrt{\nu\mu})$ is given by

$\lambda_{1,2}=\frac{1}{2}(2-\nu\pm\sqrt{(2-\nu)^2-4(1-\nu+\nu\mu)})$. So, the eigenvalues are real if

$(2-\nu)^2-4(1-\nu+\nu\mu)>0 \quad \Rightarrow \nu^2-4\nu\mu>0.$

If $\nu^2-4\nu\mu>0,\ \mu>\frac{\nu-2}{\nu}, where\ \nu\neq 0$ the map (1.1) undergoes a pitchfork bifurcation at

$(-\mu,\sqrt{\nu\mu})$.

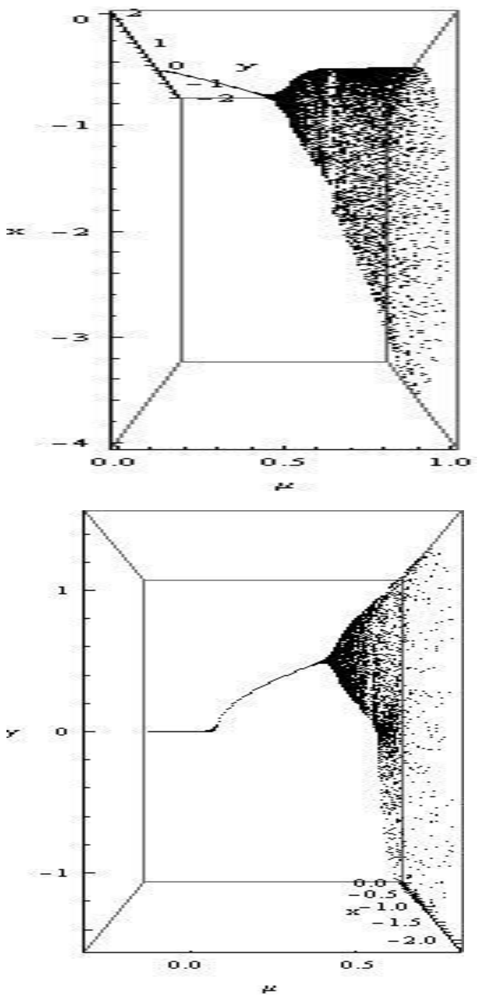


Figure 1: Pitchfork bifurcation diagram with $\nu=1$ *in* $(\mu-x)$ *and* $(\mu-y)$ *plane covering range* $\mu\in[0,0.99]$

3.3 Flip bifurcation at (0,0):

By Schur-Cohn criterion [8],[12],[16],[17],[18],[19],[20], for the map, flip bifurcation occurs if

(a) $A(1)>0$ and $A(-1)=0$.

(b) $D_1^{\pm}>0$

Therefore from (3.2), we get $A(-1) = 4 + 2\mu - 2\nu - \nu\mu = 0 \Rightarrow \mu = -2$ and $A(1) = -\nu\mu$.

Now, for $\mu = -2$, $A(1) = 2\nu > 0$ if $\nu > 0$.

Again, $D_1^{\pm} = 1 \pm a_0 = 1 \pm (1 + \mu - \nu - \nu\mu) = 2 + \mu - \nu - \nu\mu$ and $\nu + \nu\mu - \mu$.

Now, $D_1^{\pm} > 0$, if $2 + \mu - \nu - \nu\mu > 0$ and $\nu + \nu\mu - \mu > 0$.

$\Rightarrow \nu > 0$ and $\nu < 2$ for $\mu = -2$.

Again, **Criterion: -** Flip bifurcation occurs when one of the eigenvalue becomes -1.

If $\mu = -2$, the jacobian matrix at (0,0) has eigenvalues $\lambda_1 = 1 - \nu$ and $\lambda_2 = -1$.

when $\mu = -2$; $0 < \nu < 2$; the map undergoes a flip bifurcation at (0, 0).

3.4 Flip Bifurcation at $(-\mu, \sqrt{\nu\mu})$

By Schur-Cohn criterion, for the map, flip bifurcation occurs if

(a) $A(1) > 0$ and $A(-1) = 0$.

(b) $D_1^{\pm} > 0$

Therefore from $A(\lambda) = \lambda^2 + \lambda(\nu - 2) + (2\nu\mu - \nu + 1)$, we get

$A(-1) = 2 - \nu + \nu\mu = 0 \Rightarrow \mu = \dfrac{\nu - 2}{\nu}, \nu \neq 2;$ and

$A(1) = 2\nu\mu > 0 \Rightarrow \nu > 0, \mu > 0$ or $\nu < 0, \mu < 0$;

Now, for $\mu = \dfrac{\nu - 2}{\nu}$, $A(1) = 2\nu\mu = 2\nu\left(\dfrac{\nu - 2}{\nu}\right) = 2\nu - 4 > 0 \Rightarrow \nu > 2$.

(I)

Again, $D_1^{\pm} = 1 \pm a_0 = 1 \pm (1 - \nu + 2\nu\mu) = 2 + 2\nu\mu - \nu$ and $\nu - 2\nu\mu$.

Now, $D_1^{\pm} > 0$, if $2 + 2\nu\mu - \nu > 0$ and $\nu - 2\nu\mu > 0$.

(II)

Also, (I) and (II) implies $\nu > 2\nu\mu$ and $\nu > 2 \Rightarrow \nu^2 - 4\nu\mu > 0$.

Again, Flip bifurcation [9] occurs when one of the eigenvalue becomes -1.

And, for flip bifurcation to take place, one of the eigenvalues must be -1.So, if we take $\lambda_2 = -1$, then the above equations (3.3) & (3.4) reduces to

$\lambda_1 = -1 + \nu - \mu + \nu\mu - x + \nu x - 2y^2$

(3.8)

$\lambda_1 = 3 - \nu + \mu + x$

(3.9)

Equating (3.8) and (3.9) ,we get $-1+\nu-\mu+\nu\mu-x+\nu x-2y^2=3-\nu+\mu+x$

(3.10)

Now, putting $x=-\mu, y=\sqrt{\nu\mu}$ in equation (3.10) and solving for μ, we get $\mu=\dfrac{\nu-2}{\nu}$.

Also the eigenvalues of the Jacobian matrix at the point $(-\mu,\sqrt{\nu\mu})$ is given by

$\lambda_{1,2}=\dfrac{1}{2}(2-\nu\pm\sqrt{(2-\nu)^2-4(1-\nu+\nu\mu)})$.

So, the eigenvalues are real if $(2-\nu)^2-4(1-\nu+\nu\mu)>0 \quad \Rightarrow \nu^2-4\nu\mu>0.$

If $\nu^2-4\nu\mu>0$; $\mu=\dfrac{\nu-2}{\nu}$ and $\nu>2$, the map (1.1) undergoes a flip bifurcation at

$(-\mu,\sqrt{\nu\mu})$.

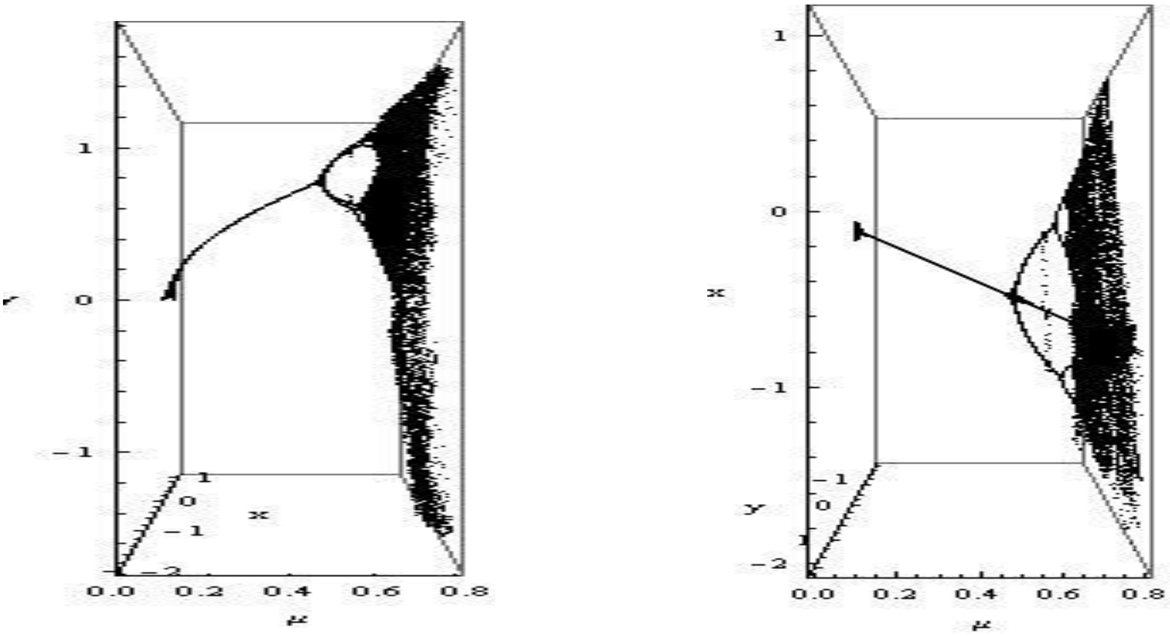


Figure2: Flip Bifurcation diagram in $(\mu-x)$ and $(\mu-y)$ plane for $\nu=2.01$;

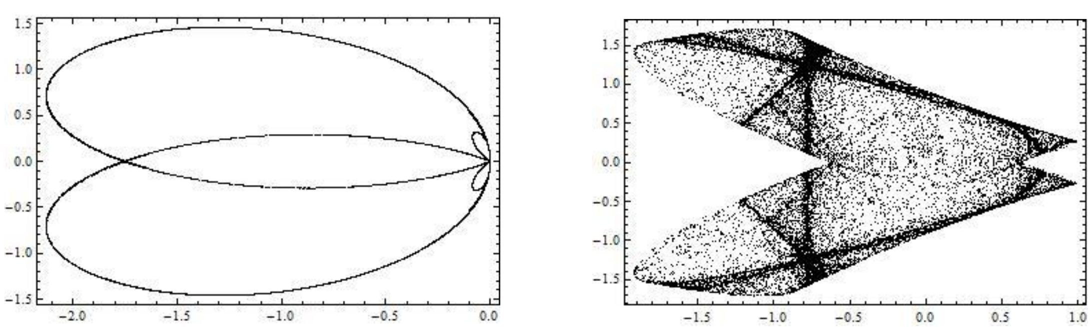


Figure 3 : Phase portrait when $\mu=0.78,\nu=1$ & $\mu=0.78,\nu=2.01$ respectively.

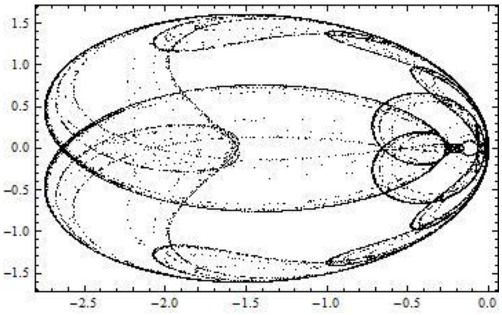


Figure 4 : Strange attractor when $\nu = 0.9\,, \mu = 0.856\,;$

4. CONTROLLING CHAOS:

The above system exhibits a strange attractor for $\nu = 0.9\,, \mu = 0.856\,;$

It has been observed that for such parameter values the fixed point (-0.856, 0.87772433) of the system becomes unstable and we desire to stabilize it .Also, the map is unpredictable at the neighboring point. [16],[21]

Now set $\mu = 0.812$ and allow the control parameter ,in this case ν ,to vary around a nominal value, say, $\nu_0 = 0.91$,for which the map has a chaotic attractor. In this particular case, the fixed points of period one are located approximately at $A = (x_{1,1}, y_{1,1}) = (\ 0.812, 0.859605)$ and $B = (x_{1,2}, y_{1,2}) = (-0.812, -0.859605)$. The Jacobian matrix of partial derivatives of the map is given by $J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix}$

Where $\begin{matrix} P(x, y) = (1-\nu)x - y^2 \\ Q = y + \mu y + xy \end{matrix}$. Thus $J = \begin{pmatrix} 1-\nu & -2y \\ y & 1+\mu+x \end{pmatrix}$

Consider the fixed point at A; the fixed point is a saddle point

4.1 The OGY Method:

Consider the n-dimensional map $z_{n+1} = f(z_n, p)$

(4.1)

Where p is some accessible system parameter that can be changed in a small neighbourhood of its nominal value, say, p_0 .In the case of continuous-time systems ,such a map can be constructed by introducing a transversal surface of section and setting up a Poincare map.[16]

It is eminent that a chaotic attractor is densely filled with unstable periodic orbits and that ergodicity guarantees that any little district on the chaotic attractor will be visited by a chaotic orbit .The OGY method relies upon the nearness of stable manifolds around unstable periodic points. The fundamental thought is to make little time-dependent linear perturbations to the control parameter p so as to push the state towards the stable manifold of the desired fixed point. Note this must be practiced if the orbit is in a little neighborhood or control locale of the fixed point. [16],[18],[19],[20],[22]

Suppose that $z_s(p)$ is an unstable fixed point of equation (4.1). The position of this fixed point moves smoothly as the parameter p is varied. For values of p close to p_0 in a small neighbourhood of $z_s(p_0)$, the map can be approximated by a linear map given by [7],[14],[15],[6]

$$z_{n+1} - z_s(p_0) = J(z_n - z_s(p_0)) + C(p - p_0)$$

(4.2)

Where $z_n = (x_n, y_n)^T$, $A = z_s(\alpha_0), J$ is the jacobian, and $C = \begin{pmatrix} \frac{\partial P}{\partial \alpha} \\ \frac{\partial Q}{\partial \alpha} \end{pmatrix}$ and all partial derivatives are evaluated at α_0 and $z_s(\alpha_0)$ .Assume in a small neighbourhood of A ,

$\alpha - \alpha_0 = -K(z_n - z_s(\alpha_0))$,

where $K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$

Substitute (5.2) into (5.1) to obtain $z_{n+1} - z_s(\alpha_0) = (J - CK)(z_n - z_s(\alpha_0))$

Therefore, the fixed point at $A = z_s(\alpha_0)$ is stable if the matrix $J - CK$ has eigenvalues with modulus less than unity. In this particular case, $J - CK \approx \begin{pmatrix} 0.09 - 0.812k_1 & -1.71921 - 0.812k_2 \\ 0.859605 & 1 \end{pmatrix}$

And the characteristic polynomial is given by

$$\lambda^2 + \lambda(0.812k_1 - 1.09) + (0.697999k_2 - 0.812k_1 + 1.56784)$$

Suppose that the eigenvalues (regular poles) are given by λ_1 *and* λ_2; then

$$\lambda_1 \lambda_2 = 0.697999k_2 - 0.812k_1 + 1.56784 \ .$$

(4.3)

$$-(\lambda_1 + \lambda_2) = 0.812k_1 - 1.09$$

(4.4)

The lines of marginal stability [16] are determined by solving the equations

$\lambda_1 = \pm 1$ *and* $\lambda_1 \lambda_2 = 1$.These conditions guarantee that the eigenvalues λ_1 *and* λ_2 have modulus less than unity.[13],[15]Suppose that $\lambda_1 \lambda_2 = 1$, then from (4.3) we get

$1 = 0.697999k_2 – 0.812k_1 + 1.56784$ (4.5)

Therefore, $k_2 = -1.43267(0.56784 - 0.812k_1)$

If $\lambda_1 = 1$, then from (4.3) we get $\lambda_2 = 0.697999k_2 - 0.812k_1 + 1.56784$

(4.6)

And from (4.4) we get $-(1+\lambda_2) = 0.812k_1 - 1.09 \Rightarrow \lambda_2 = -0.812k_1 + 1.09 - 1$

(4.7)

Therefore, $k_2 = -2.11725$

If $\lambda_1 = -1$, then from (4.3) we get $-\lambda_2 = 0.697999k_2 – 0.812k_1 + 1.56784$

(4.8)

and from (4.4) we get $-(-1+\lambda_2) = 0.812k_1 - 1.09 \Rightarrow \lambda_2 = -0.812k_1 + 1.09 + 1$

(4.9)

Therefore, $k_2 = -1.43267(3.65784 - 1.624k_1)$

The stable eigenvalues (regular poles) lie within a triangular region as depicted in Figure 5.

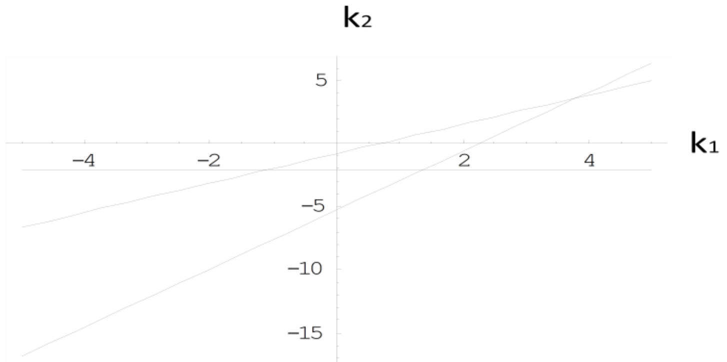


Figure 5 : The bounded region where the regular poles are stable.

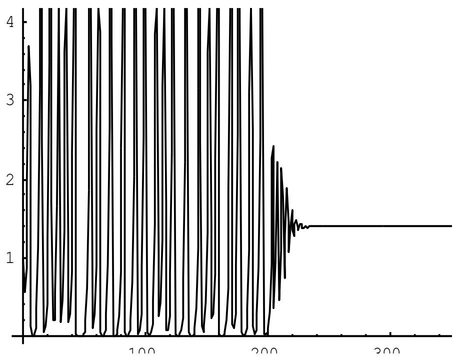
Let us select $k_1 = 1.5$ and $k_2 = 0.4$ .This point lies well inside the triangular region as depicted in above

figure 5. The perturbed Burger's map becomes [6], [7],[14],[15],[16],

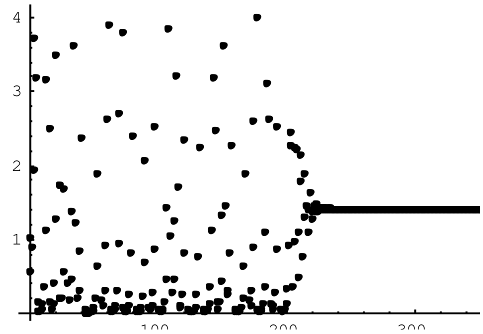
$x_{n+1} = (k_1(x - x_{1,1}) + k_2(y - y_{1,1}) + 1 - a)x - y^2$, $\quad y_{n+1} = (1+b)y + xy$;

(4.10)

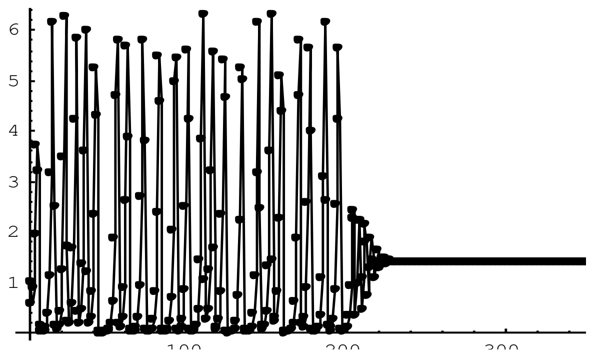
Applying equations (1.1) and (4.10) without and with control , respectively,it is possible to plot time series data for these maps. Figure below shows a time series plot when the control is switched on after the 200th iterate;



(a)



(b)



(c)

Figure 6 (a to c).Time series data for the Burgers map with and without control,
$r^2 = x^2 + y^2$*.In this case the control is activated after 200th iteration.*

5. CONCLUSIONS:

The main aim of the paper is to investigate various bifurcations in Burgers Mapping with stability analysis by algebraic methods (Schur-Cohn criterion). Pitchfork bifurcation and flip bifurcation is discussed here. There are some other bifurcations such as neimark Sacker bifurcation also which can be examined. Theoritical results are justified by the bifurcation diagrams. Also chaos is controlled with OGY method.

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