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ENHANCING THE PRODUCTION IN PROCESSIN A BAKERY USING OPTIMIZATION TECHNIQUES

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ABSTRACT

The aim of an XYZ bakery operating in Mumbai is to make profit as that is what guarantees its continuous existence and productivity in this market. In this industrial era, a bakery at all levels is facing challenges of producing goods of right quality, quantity and at right time. Also maintaining the products' shelf life at minimum cost and maximum profit for their survival and growth is of utmost important to earn more profit. Thus, this demands an increase in productive efficiency of the business. The objective of this work is to use linear programming (LP) to build a mathematical model to maximize the income of the bakery. In this study, the algorithm of Integer Programming Method was used to allocate raw materials (cupcakes, pastries and cakes) as variables in the bakery for the purpose of maximizing the income.

KEYWORDS:

Linear Programming Method, Integer Programming Problem, Maximization
Optimization
Simplex method

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1. INTRODUCTION

Linear programming (abbreviated as LP, also called as linear optimization) is a technique for optimizing the outcome (such as maximizing profit or minimizing the cost) in a form of a mathematical model. The requirements of the study are represented in the form of linear relationships. "Optimization" comes from the same root as "optimal", which means best. Linear Programming can be defined as a mathematical technique that is used to determine the best allocation of a firm's or organizations limited resources to achieve optimum goal. Here the optimum goal can be either maximizing profit or reducing the man hours or labour. The different types of Optimization problems are:

- Some problems have constraints and some do not.
- There can be one variable or more variables in the study.
- The value of the variables can be discrete (for example, only have integer values) or continuous. In case of discrete values, we use Integer Programming.
- Some problems can be static which do not change over time while some are dynamic for which continual adjustments must be made as changes occur.
- Systems can be or stochastic.

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A LP model is generally used for designing and solving the constraints to determine the best course of action.
The objective function may be of maximization of profit or minimization of costs or man hours or to reduce the raw materials that are used in the study since some raw materials are rare. Moreover, the model consists of certain structural constraints which are set of conditions. These set of conditions should justify the optimal solution. An optimum solution fulfills both the constraints of the problem that are the linear functions or linear equations and the set objective function to be met. The values of the variables in a problem are limited is defined by the constraints. Constraints are inequalities. One of the assumptions for linear programming techniques to work is that all the constraints should be linear inequalities. The region that is bounded by the system of constraints is called as the feasible region. The possible values of the variables that satisfy the constraints are represented by the feasible region.

Linear programming is a method that is used for optimizing a linear objective function, subject to linear equality and inequality constraints. Linear programs are problems that can be expressed in canonical form as:

$$Maximize: c^T x$$

$$subject\ to: Ax \leq B \text{ and } x \geq 0$$

Where, *x* denotes the vector of variables (to be determined), *c* and *b* are vectors of coefficients, *A* is a matrix of coefficients. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The objective function is a function that defines some quantity that should be minimized or maximized. The inequalities $Ax \leq b$ and $x \geq 0$ are the constraints. Two vectors are comparable when they have the same dimensions in linear programming.

Linear programming applications can be found in various fields of study and it is widely used in mathematics, business, economics, and for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It can also be useful in modeling diverse types of problems in the real world which require planning, routing, scheduling, assignment, and design. For our analysis we use Integer Programming Problem.

An integer programming problem is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. It is also known as integer linear programming (ILP), in which the objective function and the constraints (other than the integer constraints) are linear. It is said to be a mixed integer program when some variables are restricted to be integer, and is called a pure integer program when all decision variables are integers. ILP is widely used in all mathematical programming. Some applications of Integer Programming Model are Capital Budgeting, Warehouse Location, Scheduling, etc.

Sensitivity Analysis is an approach that is used for finding out the amount by which we can change the input data for the output of our linear programming model to remain comparatively unchanged. In other words, sensitivity analysis is used to see how much the outcome of the study or objective function changes when the inputs are changed. This helps us in determining the sensitivity of the data. If a small change in the input or raw materials produces a large change in the optimal solution for some models, and doesn't affect the optimal solution as much for other model, we can conclude that the second problem is more robust then the first. The second model is less sensitive to the changes in the input data. Sensitivity analysis can be applied when there are one or more input variables.

In Python sensitivity report provides us with information given below:

- Slack Value
- Shadow Price

2. CASE STUDY

The aim of every manufacturing industry is to make profit that will guarantee its continuous existence and productivity. In this industrial era especially after the industrial revolution, bakery industries are faced with the more and more challenges. The challenges are of producing goods of right quality, quantity and at right time and at minimum cost and maximum profit for their survival and growth. The increase in different brands competing for profits and a place in the market has led to increase in use of optimization techniques for the sustainability of a bakery in the market. Thus, this urges for an increase in productive efficiency of the industry. Production of bakery goods is strictly time sensitive due to the yeast proofing of dough. In that case, special requirements for production planning and scheduling, which is in bakeries often completely based on the practical experience of the responsible employee instead of mathematical methods. This often leads to suboptimal performance of companies due to the sometimes “chaotic” scheduling approach. In the project we have considered an XYZ Bakery operating in Mumbai. The bakery offers a variety of items ranging from cakes, pastries to puffs and sandwiches. In the study, only the production of desserts which are mainly cakes, cupcakes and pastries is considered. This was done as the material required for making desserts and sandwiches are different and also the customers would be different. The products like sandwiches and cupcakes can be purchased together and hence, there is a scope of market basket analysis is possible to see if a consumer buying some type of dessert also buys sandwiches or puffs.

3. LITERATURE REVIEW

Researchers, in the past have conducted a lot of study on integrated solutions for solving Optimization problem using Simplex method.

Balogun, Jolayemi, Akingbade and Muazu (2012) applied LP technique to find the optimal production process for Coca-Cola Company. To formulate the Linear Programming model, nine decision variables were considered which were the products produced by the company such as Coke, Fanta, Fanta tonic, Krest soda etc. The concentration of the drinks, sugar content, water volume and carbon (iv) oxide present in each of the drinks were taken as the constraints. To solve the LP problem, Simplex Algorithm was used and after the analysis, it was concluded that among the nine products the company produced, two products contributed the most to the company’s profit. Hence, the company should concentrate on the production of those two products to minimize cost.

Molefe (2017) had considered the entire planning activity in a bread producing firm as an integrated process involving several functions. In this study, the bakery was a supplier of pita bread and wraps. The bakery supplies major retail stores. The bread has a fixed shelf life which can be calculated by a number of scheduling periods. The main objective was to find whether the production process is capable of producing the products and yielding maximum profit. Hypothesis was tested to find the correlation between the production process rate and the production time to yield maximum profit. Also, optimum time required for transportation is also calculated. It was concluded that with the solution, the baking production and distribution scheduling is working well and hence are independent to each other.

Oladejo, Abolarinwa, Salawu and Lukman (2019) for their study for optimizing the profits had considered five different types of breads produced by a certain bakery in Ghana. The types of bread, cost and selling price with profits are included in the study. It was examined that various types, quantities and cost of the production in the bakery was formulated using AMPL software. The solution revealed that a certain type of bread should be produced in less quantity and hence forth its production must be stopped in order to achieve maximum profit.

Similarly, Abiodun and Clement (2017) in their study optimized the production of bread in Rufus Giwa Bakery, Nigeria, using LP technique. The bakery manufactures three types of bread such as medium bread, large bread and extra-large bread. LINGO software version 15 was used to analyse the data. Optimal solution was attained at X3 (Extra Large Bread) =1.175 and Zmax = 47572.28. Thus the bakery should stop manufacturing medium and large bread and make 235 pieces of extra-large bread only from 1.175 unit (i.e. 1bag of flour) per day to make a max profit of $239.19 per day or the unit profit on the medium bread and large bread must be increased to $198.48 and $188.30 respectively, before it becomes economical to produce.

JumaMakwebaRuteri (2013) stated that as any other business, food sector industry also needs some innovation to stay ahead in the competition. No business can continue to offer the same product that remains unchanged forever. The main aim of applying Linear Programming subject to the constraints that were given was to generate high profit by comparing the profits of tomato juice, tomato ketchup and tomato sauce. The solution indicates that Tomato ketchup generates more profit than the other two products, so the production of Tomato Ketchup should be increased.

TitilayoDorcasAilobhio, AlhajiIsmailaSulaiman and Imam Akeyede (2018) allocated scarce resources to maximize the profit or minimize the cost. The study was carried out to arrive at an optimal solution in the

Lace Baking Industry Lafia, Nigeria, for the production of 6 different types of bread packages. The formulated mathematical problem was solved using the R software. The solution revealed that two types of bread should be produced and sold so as to maximize the profit and the production of the remaining four types of bread should be stopped.

Akpan, N.P. &Iwok, I.A. (2016) applied the Simplex Algorithm for allocating raw materials to the competing products, small loaf, big loaf and giant loaf in a bakery with an aim of maximizing the profit. From the analysis, it was derived that small loafs primarily contributed to the profit, followed by big loafs and giant loafs did not contribute to the profit. Hence, in order to maximize the profit, larger quantities of small loafs and big loafs are to be produced and sold and the production of the giant loaf can be reduced or stopped.

EzemaandAmaken (2012) used the technique of linear programming for the purpose of profit optimization in golden plastic industry to arrive at an optimal product mix. To achieve this, a linear programming problem was formulated with respect to the production of plastic. In the scenario under consideration, polyvinyl chloride pipes of eight different sizes were being produced by the company initially. The analysis shows that only two sizes of the polyvinyl chloride pipes should be produced for profit maximization.

4. LINEAR PROGRAMMING MODEL

The problem of linear programming may be expressed as that of the optimization of linear objective function of the following type:

$Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$

Subject to the constraints of the form:

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n\ (\leq \text{ or } \geq)\ b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n\ (\leq \text{ or } \geq)\ b_2$

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots + a_{mn}x_n\ (\leq \text{ or } \geq)\ b_m$

where $x_1, x_2, \ldots\ldots x_n \geq 0$

Here $c_1, c_2, \ldots\ldots c_n$ represent the per unit profit of the decision variables $x_1, x_2, \ldots\ldots x_n$.

$a_{11}, a_{12}, \ldots a_{1n}, \ldots. a_{m1}, a_{m2} \ldots a_{mn}$ represent the amount of resources consumed per unit of the decision variables and b_i represents the total availability of the ith resource.

- Standard Form of Linear Programming Problem

For solving the Linear Programming Program using the simplex method we need to convert the problem into its standard form. The problem can be converted in the standard form using the following:

1. All the inequalities should be converted to equalities by adding or subtracting the variables. The variables which are added on the left hand side are called slack variables and variables which are subtracted from the left hand side are called surplus variables.

2. The RHS of the constraints should be non-negative, if not should be multiplied with -1 throughout.

3. All variables must have non-negative values.

4. The objective function should always be of maximization.

For n decision variables and m constraints, the standard form of the linear programming model can be formulated as follows,

$Z = c_1x_1 + c_2x_2 + \ldots\ldots + c_nx_n + 0.S_1 + 0.S_2 + \ldots + 0.S_n$

Subject to the linear constraints of the form:

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_{1n}x_n + S_1 = b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots\ldots + a_{2n}x_n + S_2 = b_2$

$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots\ldots + a_{mn}x_n + S_m = b_m$

where $x_1, x_2, \ldots x_n, S_1, S_2, \ldots S_m \geq 0$

- Assumptions

1. The raw materials required are limited.
2. The effective allocation of raw materials will aid optimal production and at the same time maximizing the profit.
3. The qualities of raw materials used in cake production are standard.
4. Some of the raw materials used in cake production cannot be stored for a long duration.

DATA PRESENTATION

The data for this study was collected from an XYZ bakery located in Mumbai, India. The data consists of total amount of raw materials that are available for daily production of cupcakes, pastries and cakes. The raw materials that are included in our study are sugar, flour, cocoa powder, eggs, crèmes and butter. The content of each raw material for one unit of each type of cake produced is shown below:

- Flour

Total amount of flour available = 25000g
Each unit of cupcake requires 10g of flour
Each unit of pastry requires 10g of flour
Each unit of cake requires 175g of flour

- Cream

Total amount of Cream available = 5000ml
Each unit of cupcake requires 0ml of Cream
Each unit of pastry requires 25ml of Cream
Each unit of cake requires 150ml of Cream

- Eggs

Total amount of Eggs available = 23000g
Each unit of cupcake requires 7g of Eggs
Each unit of pastry requires 40g of Eggs
Each unit of cake requires 730g of Eggs

- Butter

Total amount of Butter available = 10000g
Each unit of cupcake requires 10g of Butter
Each unit of pastry requires 15g of Butter
Each unit of cake requires 175g of Butter

- Sugar

Total amount of Sugar available = 20000g
Each unit of cupcake requires 10g of Sugar
Each unit of pastry requires 10g of Sugar
Each unit of cake requires 175g of Sugar

- Cocoa Powder

Total amount of Cocoa Powder available = 5000g
Each unit of cupcake requires 8g of Cocoa Powder
Each unit of pastry requires 13g of Cocoa Powder
Each unit of cake requires 40g of Cocoa Powder

- Profit contribution per unit product produced

Each unit of cupcake = N40
Each unit of pastry = N75
Each unit of cake = N400

The above data can be summarized in a tabular form given below:

TABLE I. DATA TABLE

Raw Material	**Product**			**Total Availability**
	Cupcake	**Pastry**	**Cake**	
Cream	0	25	150	5000
Butter	10	15	175	10000
Eggs	7	40	730	23000
Sugar	10	10	175	20000
Flour	10	10	175	25000
Cocoa Powder	8	13	40	5000
Revenue	**40**	**75**	**400**	

5. ANALYSIS & CONCLUSION

The data was analyzed by using the library pulp in python. The simplex method is used for the analysis. The answer report shows the optimum values for all constraints. Following is the snapshot of the constraints and the objective function.

FIGURE I. OBJECTIVE FUNTION & CONSTRAINTS

```
Problem:
MAXIMIZE
40*x_1 + 75*x_2 + 400*x_3 + 0
SUBJECT TO
_C1: 25 x_2 + 150 x_3 <= 5000

_C2: 10 x_1 + 15 x_2 + 175 x_3 <= 10000

_C3: 7 x_1 + 40 x_2 + 730 x_3 <= 23000

_C4: 10 x_1 + 10 x_2 + 175 x_3 <= 20000

_C5: 10 x_1 + 10 x_2 + 175 x_3 <= 25000

_C6: 8 x_1 + 13 x_2 + 40 x_3 <= 5000

VARIABLES
0 <= x_1 Integer
0 <= x_2 Integer
0 <= x_3 Integer
```

TABLE II. OBJECTIVE (MAX)

Name	**Final Value**
Revenue	30425

The optimal solution to the LPP has the value 30425 and the decision variables are, X_1 = 430, X_2 = 43, X_3 = 25.

SENSITIVITY ANALYSIS

The sensitivity analysis tells us how robust our solution is to the input data. The sensitivity analysis tells us how the optimal solution changes when we change the coefficients of the model. Following is the sensitivity report for the analysis:

FIGURE III. SENSITIVITY ANALYSIS

	name	shadow price	slack
0	_C1	-0.0	175.0
1	_C2	-0.0	680.0
2	_C3	-0.0	20.0
3	_C4	-0.0	10895.0
4	_C5	-0.0	15895.0
5	_C6	-0.0	1.0

The slack variable gives the units of raw materials for that constraint are under-utilized. The slack for Cream is 175, Butter is 680, Eggs is 20, Sugar is 10895, Flour is 15895 and Cocoa Powder is 1. This signifies that Cream, Eggs and Cocoa Powder were efficiently utilized but Sugar and Cream was under-utilized. So, the units of Sugar and Flour can be decreased. Since, for all the constraints the shadow price is zero. Shadow price gives us by how much the optimal value will change if the total availability of the raw materials is changed. In the study, the shadow price is zero, it means that the total availability of the raw materials should not be increased.

Conclusion

Thus, from the experiment, it was observed that it is optimal to produce 430 units of Cupcakes, 43 units of Pastries followed by 25 units of Cakes which provides a maximum income of Rs.30425. Also, for the bakery to obtain maximum income, they should produce more units of Cupcakes followed by Pastries.

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