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PROBABILISTIC AND DETERMINISTIC APPROACHES IN MEASURING TECHNICAL EFFICIENCY OF SCHOOLS IN COIMBATORE DISTRICT

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ABSTRACT

The main aim of this paper is to estimate the Technical Efficiency of the schools in Coimbatore district using probabilistic Normal Half-Normal Stochastic Frontier Production Model (NHNSFPM) and the deterministic Linear Programming Data Envelopment Analysis model. The present study incorporates Stochastic Frontier Analysis and Data Envelopment Analysis techniques to measure the technical efficiency of the three sectors of schools namely government, private and aided schools at their secondary and higher secondary levels on the basis of their performance and views on the Mathematics and Science subjects and school related factors viz., student-teacher ratio(STR), socio-economic status(SES),syllabus(SYL), learning disability(LD), teaching related factors(TF) and school facilities(SF). The primary data was collected from all the three sectors of schools in Coimbatore district, Tamil Nadu. The study also identifies the key variable and the sub-factors with respect to the key variable affecting the efficiency of the schools among the seven input variables considered for the study.

KEYWORDS:

Technical Efficiency,
Normal Half-Normal Stochastic Frontier Production Model,
Translog Production Function,
Cobb-Douglas Production Function,
DEA

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INTRODUCTION

PRODUCTIVITY

Productivity of a firm is a natural measure of performance which can be defined as the ratio of outputs to inputs, where the larger values of the ratio is associated with better performance and Efficiency can be regarded as the highest productivity level from each input level.

$$Productivity = \frac{Total\ Output}{Total\ Input} = \frac{\sum_{j=1}^{m} y_j}{\sum_{i=1}^{n} x_i}$$

where y_j $(j = 1,2, \dots, m)$ refers to the set of outputs produced, x_i $(i = 1,2, \dots, n)$ refers to the set of inputs.

PRODUCTION FUNCTION

The functional relation between the quantities of inputs used by the firm and the quantity of output produced by it, is known as the production function, which indeed reflects the firm's technology.

The mathematical formulation of a production function can be described as follows:

Let $X = (x_1, x_2, \dots, x_N)$ be the vector of N inputs and y be an output. (Kumbhakar and Lovell, 2003)

$$y = f(x_1, x_2, \dots, x_N, H)$$

where $x_1, x_2, \dots, x_N$ are the inputs, y is the output and H is the technology adopted.

PRODUCTION FRONTIER

The production frontier of the i[th] firm producing a single output with multiple inputs following the best practice technique can be defined as

$$y_i^* = f(x_{i1}, x_{i2}, \dots, x_{in} | H)$$

where y_i^* and $x_i's$ are the frontier output and inputs of the i[th] firm respectively and H is the given technology.Producers operating on their production frontier are called technically efficient, and the producers operating beneath their production frontier are called technically inefficient.

STOCHASTIC FRONTIER PRODUCTION MODELS

Coelli*et al.*, (2005) assumed a functional form for the relationship between inputs and an output in the Stochastic Frontier Analysis method of frontier estimation

$y_i = f(x_i, \alpha) + \varepsilon_i$ where $\varepsilon_i = v_i - u_i$

where x_i- vector of n inputs used by producer i;y_i- scalar output of producer i;

$f(x_i, \alpha)$- production frontier; α-vector of parameters to be estimated; u_i −two-sided error component andv_i- non-negative technical inefficiency component.

Two production functions namely Cobb-Douglas production function and Translog production functions are considered in the present study.

The general form of Cobb-Douglas production function is

$$ln y_i = \alpha_0 + \sum_n \alpha_n ln\, x_{ni} - u_i$$

and that of Translog production function is

$$\ln y = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln\, x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{i=1}^{n} \alpha_{ii}\,(\ln x_i)^2 + \sum_{i=1}^{n}\sum_{j=1}^{m} \alpha_{ij}\,(\ln x_i) * (\ln x_i)$$

TECHNICAL EFFICIENCY

The Technical Efficiency of a firm is defined as the ratio of the observed output to the maximum feasible output.

The Technical Efficiency TE_i of a producer 'i' is given by

$$TE_i = \frac{y_i}{f(x_i, \alpha) exp\{v_i\}}$$

where y_i- scalar output of producer i; $f(x_i, \alpha)$- production frontier, α – vector of parameters to be estimated and v_i-non-negative technical inefficiency component.

Technical Efficiency TE_i can be attained by the exponential conditional expectation of a two- sided error component u_i given the composed error term $\in_i$, which is given by

$TE_i = exp\left[-E\left({}^{u_i}/_{\in_i}\right)\right]$ as suggested by Johndrow *et al.*,(1982).

The output-oriented Technical Efficiency refers to the ability to obtain maximum output from a given input vector.

STOCHASTIC FRONTIER ANALYSIS

Stochastic frontier models were first developed by Aigner, Lovell and Schmidt (1977) and Meeusen and Van den Broeck (1977). The key perspective of Stochastic Frontier Analysis is the introduction of the composite error term which contains two components, a technical inefficiency component and a noise component. A firm is said to be efficient or inefficient with respect to its own production frontier based on the composite error term.

DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis is a linear programming-based technique used for measuring the efficiency of the firms with respect to certain influencing factors. The principles of DEA date back to Farrel(1957), followed by a series of discussions and comprehensive studies by Charnes, Cooper and Rhodes(1978) followed by Banker, Charnes and Cooper (1984).The Technical Efficiency of the nth Decision-Making Unit(DMU) is given by the following fractional program

$$\max \frac{\sum_{i=1}^{p} v_i y_{in}}{\sum_{j=1}^{m} u_j x_{jn}}$$

subject to

$$\frac{\sum_{i=1}^{p} v_i y_{ik}}{\sum_{j=1}^{m} u_j x_{jk}} \leq 1, \forall\, k = 1,2, \dots, n$$

$$v_i, u_j \geq 0\ \forall\ i, j$$

where $i = 1,2, \dots, p; j = 1,2, \dots, m; k = 1,2, \dots, n;$ y_{ik} −amount of output i produced by the kth DMU; x_{jk} - amount of input j utilized by the kth DMU; v_i-weight given to output k, u_j-weight given to input j.

TECHNICAL EFFICIENCY OF THE NORMAL HALF NORMAL STOCHASTIC FRONTIER PRODUCTION MODEL

Distributional Assumptions

The following distributional assumptions are considered in the derivation of Stochastic Production Frontier Models

(i) $v_i \sim i.i.d.N(0, {\sigma_v}^2)$

(ii) $u_i \sim i.i.d.N^+(0, {\sigma_u}^2)$, that is as non-negative half normal

(iii) v_i and u_i are distributed independently of each other and of the regressors

The Technical Efficiency of the following Stochastic Frontier Production Models are derived in the present study and the results are obtained as depicted below

The Probability density function of u is given by

$$f(u) = \frac{2}{\sqrt{2\pi}\sigma_u} exp\left\{\frac{-u^2}{2{\sigma_u}^2}\right\} \qquad (1)$$

The Probability density function of v is given by

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma_v} exp\left\{\frac{-v^2}{2{\sigma_v}^2}\right\} \qquad (2)$$

Since u and v are independently distributed, the joint density function of u and v is the product of their individual probability density functions

$$f(u,v) = f(u).f(v) = \frac{2}{2\pi\sigma_u\sigma_v} exp\left\{\frac{-u^2}{2{\sigma_u}^2} + \frac{-v^2}{2{\sigma_v}^2}\right\} \qquad (3)$$

Using the transformation, $\varepsilon = v - u$, the joint density function of u and ε is

$$f(u,\varepsilon) = \frac{1}{\pi\sigma_u\sigma_v} exp\left\{-\frac{u^2}{2{\sigma_u}^2} - \frac{(u+\varepsilon)^2}{2{\sigma_v}^2}\right\} \qquad (4)$$

The marginal density function of ε is given by

$$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du$$

$$f(\varepsilon) = \frac{2}{\sqrt{2\pi}\sigma_s} exp\left[-\frac{\varepsilon^2}{2{\sigma_s}^2}\right] \frac{1}{\sqrt{2\pi}} \int_{\frac{\varepsilon\mu}{\sigma_s}}^{\infty} exp\left(-\frac{t^2}{2}\right) dt \qquad (5)$$

$$f(\varepsilon) = \frac{2}{\sqrt{2\pi}\sigma_s} exp\left[-\frac{\varepsilon^2}{2{\sigma_s}^2}\right] \left[1 - \Phi\left(\frac{\varepsilon\mu}{\sigma_s}\right)\right] \qquad (6)$$

$$f(\varepsilon) = \frac{2}{\sigma_s} \phi\left(\frac{\varepsilon}{\sigma_s}\right) \Phi\left(-\frac{\varepsilon\mu}{\sigma_s}\right) \qquad (7)$$

Where ϕ is the density function and Φ is the standard normal cumulativedistribution.

$$TE = \exp\left[-\sigma_{s_*} \left\{\frac{\phi\left(\frac{\varepsilon_i\mu}{\sigma_s}\right)}{\left[1-\Phi\left(\frac{\varepsilon_i\mu}{\sigma_s}\right)\right]} + \frac{\varepsilon_i\mu}{\sigma_s}\right\}\right] \qquad (8)$$

ESTIMATION OF TECHNICAL EFFICIENCY OF SCHOOLS IN COIMBATORE DISTRICT WITH RESPECT TO THEIR SCORES IN MATHEMATICS AND SCIENCE IN THEIR X AND XII STANDARD

Translog Normal Half-Normal Stochastic Production Frontier Model-TNHNSFPM

In this section the Translog Normal Half-Normal Stochastic Production Frontier Model was incorporated for the study involving 35 independent variables. The Ordinary Least Squares (OLS) estimates and the MLE estimates of the parameters of TNHNSFPM which show average performance of 450 students at their secondary and higher secondary levels were presented in the Table 3 and Table 4 respectively.

Estimation of Technical Efficiency

A frequency distribution of predicted technical efficiencies within ranges of five using TNHNSFPM is depicted in Table 1

Table 1: Frequency Distribution of Student Specific Technical Efficiency Estimates Using TNHNSFPM

Efficiency Score (%)	Mathematics				Science			
	X-Standard		XII-Standard		X-Standard		XII-Standard	
	Number of students	%	Number of Students	%	Number of Students	%	Number of students	%
Below 85	-	-	-	-	-	-	-	-
85-90	80	17.78	85	18.89	70	15.56	130	28.89
90-95	120	26.67	110	24.44	127	28.22	150	33.33
95-100	300	66.67	305	67.78	303	67.33	270	60

The highest number of students were in the technical efficiency range (95-100) and no student has reported a technical efficiency below 85% both at their X and XII standard levels with respect to both Mathematics and Science subjects

Table 2: Statistical Analysis For TNHNSFPM

Subject	Correlation Analysis		Chi-Square Test of Goodness of Fit
Mathematics	X	$r = 0.558$	$\aleph^2$=1.4612
	XII	$r = 0.539$	$\aleph^2$=1.4603
Science	X	$r = 0.557$	$\aleph^2$=1.4401
	XII	$r = 0.549$	$\aleph^2$=1.4365

Table 3: Ordinary Least Square Estimates Using TNHNSFPM

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
Constant	α_0	270.598	272.625	264.125	256.958
ln STR	α_1	-36.616	-40.758	-36.522	-36.616
ln SES	α_2	-9.258	-11.624	-9.258	-9.512
ln SF	α_3	-5.909	-8.698	-5.909	-8.158
ln LD	α_4	0.076*	0.0982*	0.075*	0.094*
ln SYL	α_5	0.315	0.589	0.315	0.496
ln TF	α_6	3.921	6.874	3.921	6.025
ln ETC	α_7	-10.759	-14.059	-10.759	-11.928
ln STR x ln STR	α_{11}	1.989	3.997	1.989	2.637
ln SES x ln SES	α_{22}	-0.598	-0.962	-0.598	-0.912
ln SF x ln SF	α_{33}	-0.298	-0.511	-0.298	-0.425
ln LD x ln LD	α_{44}	-0.612**	-0.691**	-0.615**	-0.689**
ln SYL x ln SYL	α_{55}	-0.069	-0.097	-0.069	-0.091
ln TF x ln TF	α_{66}	-0.019	-0.201	-0.019	-0.142
ln ETC x ln ETC	α_{77}	-0.125	-0.320	-0.125	-0.321
ln STR x ln SES	α_{12}	0.699	0.965	0.699	0.910
ln STR x ln SF	α_{13}	0.090	0.991	0.090	0.114
ln STR x ln LD	α_{14}	-0.293	-0.526	-0.293	-0.501
ln STR x ln SYL	α_{15}	0.009	0.089	0.009	0.089
ln STR x ln TF	α_{16}	0.058	0.087	0.058	0.068
ln STR x ln ETC	α_{17}	1.815	1.989	1.815	1.902
ln SES x ln SF	α_{23}	0.489	0.725	0.489	0.658
ln SES x ln LD	α_{24}	0.612	0.886	0.612	0.796
ln SES x ln SYL	α_{25}	0.109	0.347	0.109	0.302
ln SES x ln TF	α_{26}	-0.395	-0.698	-0.395	-0.614
ln SES x ln ETC	α_{27}	0.216	0.496	0.216	0.523
ln SF x ln LD	α_{34}	0.110	0.999	0.110	0.099
ln SF x ln SYL	α_{35}	0.025	0.156	0.025	0.109
ln SF x ln TF	α_{36}	0.296	0.392	0.296	0.299
ln SF x ln ETC	α_{37}	-0.059	-0.075	-0.059	-0.071
ln LD x ln SYL	α_{45}	0.258	0.204	0.258	0.204
ln LD x ln TF	α_{46}	0.098	0.221	0.098	0.182

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
ln LD x ln ETC	α_{47}	-0.079	-0.009	-0.079	-0.009
ln SYL x ln TF	α_{56}	-0.058	-0.062	-0.058	-0.052
ln SYL x ln ETC	α_{57}	-0.112	-0.457	-0.112	-0.412
ln TF x ln ETC	α_{67}	-0.312	-0.509	-0.312	-0.465
*Significant at 5% level		R^2=0.702	R^2=0.715	R^2=0.685	R^2=0.690
**Significant at 1% level		N=450	N=450	N=450	N=450

Table 4: Maximum Likelihood Estimates Using TNHNSFPM

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
Constant	α_0	282.558	281.225	285.526	283.258
ln STR	α_1	-41.098	-40.125	-39.258	-40.278
ln SES	α_2	-9.258	-10.458	-8.569	-9.589
ln SF	α_3	-4.986	-5.998	-4.259	-4.289
ln LD	α_4	1.715	2.021	1.512	1.998
ln SYL	α_5	0.492	0.987	0.492	0.695
ln TF	α_6	4.659	6.876	5.581	5.912
ln ETC	α_7	-16.568	-18.211	-15.963	-15.852
ln STR x ln STR	α_{11}	2.974	4.112	3.152	3.147
ln SES x ln SES	α_{22}	-0.814	-0.912	-0.694	-0.706
ln SF x ln SF	α_{33}	-0.245	-0.419	-0.269	-0.312
ln LD x ln LD	α_{44}	-0.576*	-0.705*	-0.621*	-0.609*
ln SYL x ln SYL	α_{55}	-0.053	-0.071	-0.035	-0.048
ln TF x ln TF	α_{66}	-0.152	-0.158	-0.109	-0.206
ln ETC x ln ETC	α_{77}	-0.241	-0.329	-0.206	-0.284
ln STR x ln SES	α_{12}	0.714	0.865	0.698	0.725
ln STR x ln SF	α_{13}	0.089	0.099	0.076	0.090
ln STR x ln LD	α_{14}	-0.609	-0.724	-0.485	-0.591
ln STR x ln SYL	α_{15}	-0.069	-0.086	-0.069	-0.074
ln STR x ln TF	α_{16}	-0.031	-0.022	-0.016	-0.025
ln STR x ln ETC	α_{17}	2.152	2.918	1.968	1.999
ln SES x ln SF	α_{23}	0.586	0.729	0.425	0.587
ln SES x ln LD	α_{24}	0.625	0.674	0.486	0.654
ln SES x ln SYL	α_{25}	0.096	0.082	0.056	0.071

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
ln SES x ln TF	α_{26}	-0.369	-0.512	-0.528	-0.632
ln SES x ln ETC	α_{27}	0.632	0.712	0.561	0.648
ln SF x ln LD	α_{34}	0.209	0.325	0.186	0.198
ln SF x ln SYL	α_{35}	0.224	0.302	0.118	0.179
ln SF x ln TF	α_{36}	0.215	0.299	0.215	0.276
ln SF x ln ETC	α_{37}	-0.075	-0.089	-0.089	-0.019
ln LD x ln SYL	α_{45}	0.176*	0.211*	0.119*	0.158*
ln LD x ln TF	α_{46}	0.152	0.175	0.149	0.158
ln LD x ln ETC	α_{47}	0.163	0.186	0.128	0.178
ln SYL x ln TF	α_{56}	-0.26	-0.45	-0.25	-0.28
ln SYL x ln ETC	α_{57}	-0.214	-0.416	-0.279	-0.312
ln TF x ln ETC	α_{67}	-0.171*	-0.549*	-0.372*	-0.411*
$\lambda = \frac{\sigma_u}{\sigma_v}$		2.449*	1.7678*	2.3717*	2.4037*
$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$		0.07**	0.0812**	0.0728**	0.07810**
Log-likelihood		302.968	301.818	300.720	298.156
Estimated Variances of the underlying variables					
v		0.0007	0.0016	0.0008	0.0009
u		0.0042	0.0050	0.0045	0.0052
ε		0.0049	0.0066	0.0053	0.0061
$\gamma = \frac{Var(u)}{Var(\varepsilon)}$		0.8571	0.7576	0.8491	0.8525

*Significant at 5% level **Significant at 1% level

INFERENCES

OBSERVATIONS	**IMPLICATIONS**
Table 3: Ordinary Least Squares Estimation	
R^2=0.702 R^2=0.715	the inputs used in the model were able to depict 70% and 72% of the variations X and XII standard levels respectively with respect to Mathematics subject
R^2=0.685 R^2=0.690	the inputs used in the model were able to depict 69% of the variations both at their X and XII standard levels respectively with respect to Science subject.

OBSERVATIONS		IMPLICATIONS
Positive coefficient of the parameters $\alpha_4,\alpha_5,\alpha_6$		The inputs learning disability, syllabus and teaching related factors were allocated efficiently both at their X and XII standard levels with respect to both Mathematics and Science subjects.
Negative coefficient of the parameters $\alpha_1, \alpha_2, \alpha_3,\alpha_7$		The inputs student teacher ratio, socio-economic status, school facilities and extra tuition classes were of inefficient allocation both at their X and XII standard levels with respect to both Mathematics and Science subjects.
Table 4: Maximum Likelihood Estimation		
λ>1, $\sigma_u{}^2 > \sigma_v{}^2$ at the X and XII standard levels with respect to Mathematics and Science subjects		The dominant share of the estimated variances of the one sided error term ,u, over the estimated variance of the whole error term ⇒ the residual variation in output was associated with the variation in technical inefficiency rather than with measurement error which was associated with uncontrollable factors related to the production process.
X	γ=0.8571	The difference between the observed and frontier output was primarily due to the factors which were 86% and 76% under the control of the firms at their X and XII standard levels respectively with respect to Mathematics subject.
XII	γ=0.7576	
X	γ=0.8491	The difference between the observed and frontier output was primarily due to the factors which were 85% under the control of the firms both at their X and XII standard levels respectively with respect to Science subject.
XII	γ=0.8525	

Cobb-Douglas Normal Half-Normal Stochastic Production Frontier Model-CDNHNSFPM

The Cobb-Douglas production function model considered for the study involved a total of seven independent variables. An OLS and MLE estimates of the parameters of stochastic frontier model for the sample of 450 students is presented in Table 6 and Table 7 respectively.

Estimation of Technical Efficiency

The frequency distribution of the CDNHNSFPM in frequencies of 5 were tabulated below in table 5. The highest number of students were in the technical efficiency range(95-100) and no student has reported a technical efficiency below 80% both at their X and XII standard levels with respect to Mathematics and Science subjects.

Table 5: Frequency Distribution of Student Specific Technical Efficiency Estimates Using CDNHNSFPM

Efficiency Score (%)	Mathematics				Science			
	X-Standard		XII-Standard		X-Standard		XII-Standard	
	Number of students	%	Number of Students	%	Number of Students	%	Number of students	%
Below 80	-	-	-	-	-	-	-	-
80-85	30	6.67	27	6	32	7.11	20	4.44
85-90	70	15.56	65	14.44	68	15.11	90	20
90-95	100	22.22	104	23.11	105	23.33	120	26.67
95-100	250	55.56	254	56.44	245	54.44	220	48.89

Table 6: Ordinary Least Squares Estimates Using CDNHNSFPM

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
Constant	α_0	8.023**	8.056**	7.575**	7.989**
ln STR	α_1	-0.210*	-0.199*	-0.187*	-0.206
ln SES	α_2	0.059	0.079	0.059	0.059
ln SF	α_3	-0.007	-0.008	-0.007	-0.048
ln LD	α_4	0.282**	0.398**	0.296**	0.315**
ln SYL	α_5	0.020	0.028	0.020	0.020
ln TF	α_6	-0.029	-0.041	-0.029	-0.032
Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
ln ETC	α_7	-0.179**	-0.158**	-0.169**	-0.171**
*Significant at 5% level		R^2=0.585	R^2=0.596	R^2=0.541	R^2=0.550
**Significant at 5% level		N=450	N=450	N=450	N=450

Table 7: Maximum Likelihood Estimates Using CDNHNSFPM

Variables	Parameters	Coefficients			
		Mathematics		Science	
		X	XII	X	XII
Constant	α_0	8.996**	8.826**	8.159**	6.927
ln STR	α_1	-0.145	-0.88	-0.136	-0.156
ln SES	α_2	0.039	0.049	0.039	0.046
ln SF	α_3	0.018	0.029	0.011	0.021
ln LD	α_4	0.298**	0.268**	0.268**	0.298
ln SYL	α_5	0.019	0.021	0.019	0.029
ln TF	α_6	-0.015	-0.024	-0.016	-0.033
ln ETC	α_7	-0.146**	-0.212**	-0.146**	-0.158**
$\lambda = \frac{\sigma_u}{\sigma_v}$		2.6623	2.7576	2.53239	2.79207
$\sigma = \sqrt{\sigma_u^2 + \sigma_v^2}$		0.08579	0.09088	0.08258	0.09044
Log-likelihood		278.113	280.459	273.257	272.785
Estimated Variances of the underlying variables					
v		0.00091	0.00096	0.00092	0.00093
u		0.00645	0.00730	0.00590	0.00725
ε		0.00736	0.00826	0.00682	0.00818
$\gamma = \frac{Var\ (u)}{Var\ (\varepsilon)}$		0.87536	0.88377	0.86510	0.886308

*Significant at 5% level **Significant at 1% level

Table 8: Statistical Analysis For CDNHNSFPM

Subject	Correlation Analysis		Chi-Square Test of Goodness of Fit
Mathematics	X	$r = 0.668$	$\aleph^2$=1.6325
	XII	$r = 0.672$	$\aleph^2$=1.6692
Science	X	$r = 0.645$	$\aleph^2$=1.6215
	XII	$r = 0.656$	$\aleph^2$=1.5896

INFERENCES

<table>
<tr><th colspan="2">OBSERVATIONS</th><th>IMPLICATIONS</th></tr>
<tr><td colspan="3">Table 6: Ordinary Least Squares Estimation</td></tr>
<tr><td colspan="2">R^2=0.585
R^2=0.596</td><td>the inputs used in the model were able to depict 59% and 60% of the variations at their X and XII standard levels respectively with respect to Mathematics subject.</td></tr>
<tr><td colspan="2">R^2=0.541
R^2=0.550</td><td>the inputs used in the model were able to depict 54% and 55% of the variations at their X and XII standard levels respectively with respect to Science subject.</td></tr>
<tr><td colspan="2">Positive coefficient of the parameters α_4,α_5,α_6</td><td>The inputs learning disability, syllabus and teaching related factors were allocated efficiently both at their X and XII standard levels with respect to both Mathematics and Science subjects.</td></tr>
<tr><td colspan="2">Negative coefficient of the parameters α_1, α_2, α_3,α_7</td><td>The inputs student teacher ratio, socio-economic status, school facilities and extra tuition classes were of inefficient allocation both at their X and XII standard levels with respect to both Mathematics and Science subjects.</td></tr>
<tr><td colspan="3">Table 7: Maximum Likelihood Estimation</td></tr>
<tr><td colspan="2">λ>1, $\sigma_u^2 > \sigma_v^2$ at the X and XII standard levels with respect to Mathematics and Science subjects</td><td>The dominant share of the estimated variances of the one sided error term ,u, over the estimated variance of the whole error term
⇒ the residual variation in output was associated with the variation in technical inefficiency rather than with measurement error which was associated with uncontrollable factors related to the production process.</td></tr>
<tr><td>X</td><td>γ=0.87636</td><td rowspan="2">The difference between the observed and frontier output was primarily due to the factors which were 88% under the control of the firms both at their X and XII standard levels respectively with respect to Mathematics subject.</td></tr>
<tr><td>XII</td><td>γ=0.88377</td></tr>
<tr><td>X</td><td>γ=0.86510</td><td>The difference between the observed and frontier output was primarily due to the factors which were 87% and 89% under the control of the firms at their X and XII standard levels respectively with respect to Science subject</td></tr>
</table>

DATA ENVELOPMENT ANALYSIS-DEA

Estimation of Technical Efficiency

The indices of technical efficiency measures derived from the linear programming problem mentioned in the methodology for 450 students are summarized in Table 9, which shows that the highest number of students were in the technical efficiency range(80-85) and no student has reported a technical efficiency above 95% and below 55% both at their X and XII standard levels respectively with respect to Mathematics and Science subjects.

Table 10: Statistical Analysis For DEA

Subject	Correlation Analysis		Chi-Square Test of Goodness of Fit
Mathematics	X	$r = 0.580$	$\aleph^2$=12.5392
	XII	$r = 0.574$	$\aleph^2$=12.4725
Science	X	$r = 0.569$	$\aleph^2$=12.5098
	XII	$r = 0.558$	$\aleph^2$=12.4582

Table 9: Frequency Distribution of Student Specific Technical Efficiency Estimates Using DEA

Efficiency Score(%)	Mathematics				Science			
	X-Standard		XII-Standard		X-Standard		XII-Standard	
	Number of students	%	Number of Students	%	Number of Students	%	Number of students	%
Below 55	-	-	-	-	-	-	-	-
55-60	6	1.33	12	2.67	6	1.33	-	-
60-65	10	2.22	7	1.56	13	2.89	10	2.22
65-70	14	3.11	10	2.22	14	3.11	15	3.33
70-75	30	6.67	32	7.11	27	6	50	111.11
75-80	40	8.89	35	7.78	42	9.33	75	16.67
80-85	120	26.67	121	26.89	121	26.89	100	22.22
85-90	150	33.33	148	32.89	149	33.11	120	26.67
90-95	80	17.78	85	18.89	78	17.33	80	17.78
95-100	-	-			-	-	-	-

SUMMARY OF THE ESTIMATION

Table 11: Summary Statistics of Efficiency Estimates-Mathematics- X Standard

Statistic	TNHNSFPM	CDNHNSFPM	DEA
Mean	93.57	90.89	76.70
Minimum	87.55	82.65	58.9
Maximum	99.58	99.12	94.49

Table 12: Summary Statistics of Efficiency Estimates-Mathematics- XII Standard

Statistic	TNHNSFPM	CDNHNSFPM	DEA
Mean	93.39	90.71	75.99
Minimum	87.32	82.16	57.8
Maximum	99.46	99.26	94.18

Table 13: Summary Statistics of Efficiency Estimates-Science- X Standard

Statistic	TNHNSFPM	CDNHNSFPM	DEA
Mean	92.93	90.4	75.79
Minimum	86.62	81.92	57.76
Maximum	99.23	98.99	93.82

Table 14: Summary Statistics of Efficiency Estimates-Science- XII Standard

Statistic	TNHNSFPM	CDNHNSFPM	DEA
Mean	93.18	90.7	75.76
Minimum	86.98	81.86	57.56
Maximum	99.38	99.54	93.75

POTENTIAL OF TECHNICAL EFFICIENCY IMPROVEMENT

The present analysis focuses on the achievement of higher scores in Mathematics and Science subjects with the existing resources technology. Based on the technical efficiency of the most efficient student in each of the choosen models, the aaverage potential to increase the score in the subjects was determined using the formula

$$Potential\ for\ increasing\ the\ score\ \} = \left[1-\left(\frac{Mean\ Technical\ Efficiency\ of\ the\ system}{Maximum\ technical\ efficiency\ of\ the\ system}\right)\right]*100$$

Table 15: Increasing Technical Efficiency Potential using Various Models

Model	Mean Potential to Increase Technical Efficiency			
	Mathematics		Science	
	X	XII	X	XII
TNHNSFPM	7.35	8.24	8.04	6.10
CDNHNSFPM	7.72	8.88	8.30	8.61
DEA	15.55	14.87	18.83	19.31

CONCLUSION

The technical efficiency estimation was done with 450 school students at their X and XII standard levels in Coimbatore district of Tamil Nadu. From the results summarised in the above tables, it was clear that, the efficiency estimates using stochastic frontier models and data envelopment analysis showed that the highest mean technical efficiency was given by TNHNSFPM followed by CDNHNSFPM and DEA. The highest minimum technical efficiency was given by TNHNSFPM followed by CDNHNSFPM and DEA. The highest mean potential to increase the technical efficiency was given by DEA, CDNHNSFPM and TNHNSFPM.

Among the three sectors of the schools namely Government, Private and Aided schools the technical efficiency score was high in case of private schools (99%)followed by government schools (97%) and aided schools (94%). The input variable learning disability was identified as the key variable for technical inefficiency among the seven input variables. The key variable learning disability was identified with several other sub-factors as shift from a joint family to a neutral family(18%), change of school(12%), peer-group influence(36%), insomnia(34%), study atmosphere at home, etc., Indeed many female respondents identified their learning disability was due to the sub-factorsshift from a joint family to a neutral family and change of school. The male respondents identified the sub-factors peer-group influence, insomnia and study atmosphere at home.

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