Surface waves propagation in an anisotropic two temperature generalized thermo elasticity

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Abstract

The surface waves propagation in an anisotropic two temperature generalized thermo elasticity is studied. The governing equations are solved to obtain the general solution in x-z plane. The appropriate boundary conditions at an interface between two dissimilar half spaces are satisfied by appropriate particular solution to obtain the frequency equation of the surface wave in medium.

Keywords:Thermo elasticity;Frequency; Temperature

1. Introduction

The governing equations of two-temperature generalized magneto-thermo elasticity with hydrostatic initial stress are specialized in two dimensions and are solved for surface wave solutions. The appropriate solutions in a half-space are obtained which satisfy relevant radiation condition and boundary conditions at thermally insulated as well as isothermal surface. Boley and Tolins (1962) showed that the two temperatures T and Φ , and the strain are represented in the form of a travelling wave plus a response, which occurs instantaneously throughout the body. Chen and Gurtin (1968) and Chen et al. (1968, 1969) formulated a theory of thermoe-lasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T. Warren and Chen (1973) studied the wave propagation in the two-temperature theory of coupled thermoelasticity. Puri and Jordan (2006) discussed the propagation of harmonic plane waves in two temperature theory. Youssef (2006) formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay (2010) extended the work of Puri and Jordan (2006) in the context of the linear theory of two- temperature generalized thermo elasticity formulated by Youssef (2006). Singh and Bijarnia (2011) studied the propagation of plane waves in anisotropic two temperature generalized thermo elasticity.

The frequency equation of Rayleigh wave is obtained. The frequency equation is also reduced for limiting cases of small thermal coupling and small reduced frequency. Velocity of propagation and amplitude-attenuation factor of Rayleigh wave are computed for a numerical example. To illustrate the dependence of velocity and amplitude-attenuation factor upon two-temperature parameter, initial stress parameter thermal relaxation time and magnetic field.

2. Governing equations

Following Yousssef (2006), the governing equations for two temperature anisotropic generalized thermo elasticity with one relaxation parameter are

i) The stress strain temperature relations:

$$\sigma_{ij} = c_{ijkl} e_{kl} - \gamma_{ij} (T - \Phi_0), \quad (1)$$

ii) The displacement-strain relation:

$$e_{ij} = 1/2(u_{ij} + u_{ji}) \quad (2)$$

iii) The equation of motion:

$$\rho \ddot{u} = \sigma_{ji,j} + \rho F_i, \quad (3)$$

iv) The energy equation:

$$-q_{i,i} = \rho T_o \dot{S}, \quad (4)$$

v) The modified Fourier‟s law:

$$-k_{ij} \Phi_{,j} = q_i + \tau_0 \dot{q}_i, \quad (5)$$

vi) The entropy-strain-temperature–diffusion relation:

$$\rho s = \frac{\rho c_E}{T_0}\theta + \gamma_{ij} e_{ij}. \quad (6)$$

Here, terms γ_{ij} are the coupling parameters , T is the mechanical temperature, $\Phi_0 = T_0$ Is the Reference temperature, $\theta = T - T_0$ with $|\theta / T_0| \leq 1, \sigma_{ij}$ Is the stress tensor, e_{kl} is the strain tensor, c_{ijkl} is tensor of elastic constants, ρ is the mass density, q_i is the heat conduction vector, K_{ij} is The thermal conductivity tensor, c_E is the specific head at constant strain, u_i are the components of the displacement vector ,S is the entropy per unit mass, τ_0 the thermal relaxation time ,which will ensure that the heat conduction equation will predict finite speed of heat propagation and Φ is the conductive temperature and satisfies the relation.

$$\Phi - T = a^* \Phi_{,ii}, \tag{7}$$

Where $a^* > 0$ is the two temperature parameter.

We consider a homogeneous and transversely isotropic thermo elastic medium of an infinite extent with Cartesian coordinates system(x,y,z) which is previously at uniform temperature .We assume that medium is transversely isotropic in such a way that the planes of isotropy are perpendicular to z-axis. The origin is taken on the plane surface and z-axis is taken normally in to the medium($z \geq 0$).The surface z=0 is assumed stress free and thermally insulated. T represent study is restricted to the plane strain parallel to x-z plane, with thedisplacement vector $u = (u_1, 0, u_3)$ with the help of the equations(1)–(3),we obtain the following two components of equation of motion:

$$c_{11} u_{1,11} + (c_{13} + c_{44}) u_{3,13} + c_{44} u_{1,33} - \beta_1 T_{,1} = \rho \ddot{u}_1, \tag{8}$$

$$c_{44} u_{3,11} + (c_{13} + c_{44}) u_{1,13} + c_{33} u_{3,33} - \beta_3 T_{,3} = \rho \ddot{u}_3. \tag{9}$$

Equations (4) and (6) lead to the following heat conduction equation

$$K_1 \Phi_{,11} + K_3 \Phi_{,33} = \rho c_E (\dot{T} + \tau_0 \ddot{T}) + \beta_1 T_0 (\dot{u}_{1,1} + \tau_0 \ddot{u}_{1,1}) + \beta_3 T_0 (\dot{u}_{3,3} + \tau_0 \ddot{u}_{3,3}) \tag{10}$$

and the equation (7) becomes

$\Phi - T = a^*(\Phi_{,11} + \Phi_{,33})$.

They solved the problem numerically using a finite element method. They gave the numerical results for the temperature distribution, the displacement components and the thermal stress and illustrated graphically. The results so obtained were compared with the results predicted by the theory of generalized thermo elasticity with one relaxation time for different values of pressure.

Othman and Said studied the effect of rotation on two-dimensional problem of a fibre-reinforced thermoelastic with one relaxation time. They applied the Lord–Shulman (L–S) theory with one relaxation time and coupled theory to study the influence of reinforcement on the total deformation of a rotating thermoelastic half-space and the interaction with each other. They solved the problem of a thermal shock numerically using normal mode analysis. They gave the numerical results for the temperature, displacement, and thermal stress components and illustrated graphically for both L–S and coupled theories.Conclusion

The appropriate solutions of all the governing equations of the surface waves propagation in an anisotropic two temperature generalized thermo elasticity is studied. The governing equations for two temperature anisotropic generalized thermo elasticity with one relaxation parameter are solved to obtain the general solution in x-z plane. The appropriate boundary conditions at an interface z = 0 between two dissimilar half spaces are satisfied by appropriate particular solution to obtain the frequency equation of the surface wave in medium. Frequency equations of surface waves are also obtained for some limiting cases in absence of thermal parameters and transverse anisotropy.

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