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GRAPH THEORETICANALYSIS OF AN ELECTRICAL CIRCUIT WITH UNKNOWN RESISTANCES AND VOLTAGES

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ABSTRACT

Electrical circuits can be studied and simplified by using many well known theorems of Electrical Engineering. An electrical circuit can be represented by graph also. The circuit diagram can be converted to different matrices by using specific definitions. There are many matrix equilibrium equations which can be obtained from different laws of electrical circuits. The results of these equations can be verified by the existing theorems and graph theory. Some difficult problems become interesting and also look simple when represented using a graph. Graph theory is useful and powerful technique for solving real-world problems as it gives different kinds of optimization. It seems to be laborious to use the existing and traditional methods to find branch currents and loop currents of electrical circuits if it contains many branches. Using mathematical model this difficulty can be solved. In our previous paper we analysed currents in a circuit with all known resistances and voltages. In this paper, branch currents of an electrical circuit with unknown resistances and voltage are obtained using network equilibrium equations and then it will be verified by Thevenin's theorem.

KEYWORDS:

Graph theory;
Electrical circuits;
Matrix Representation;
Branch current;
Loop current.

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1. INTRODUCTION:

Graph Theory is a branch of Mathematics that deals in some points and the connections between them. A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$ is defined by a particular relation. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \dots . x_n\}$and edge is a line which connects two pointsx_i and x_j.

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In Network Analysis, an electrical network is defined as the collections of some interconnected electrical devices. So both the fields can be related to each other. In Electrical Engineering, Graph Theory is also used to verify the properties of different electrical circuits. Electrical circuits consist of different electrical elements. Flowing current through the circuits obeys different laws such as Kirchhoff's current laws, Kirchhoff's voltage laws etc. Graph theory comes into account in the electrical circuits in the manner by which the elements or devices are connected in the circuits.

2. LITERATURE REVIEW:

In recent times application of Graph theory in electrical circuit analysis becomes a growing trend of research. Many researchers have done remarkable work on this topic. In 1980, Vandewalle [19] studied the application of coloured branch theorem in circuit theory. In the same year Satoru Fujishige developed an algorithm to solve graph realization problem. He used the fundamental circuit matrix or graphic matrix for developing it [13]. Istvan Fary in 1983 studied and developed some algorithms on generalised circuits [12]. In 1985, Karl Gustafson studied graph and networks using vector calculus and developed some results relating to divergence and curl [15]. In 1992, Brooks, Smith, Stone and Tutte gave an expression for current and potential differences using determinant [5]. In 1997, Gunther and Hoscheck adapted ROW method for simulation of electric circuits [14]. In 2012, Li and Xuan [17] used improved adjacency matrix for calculation of distribution network flow. In the same year bond graph was used to solve an electrical model by obtaining the system equations [18]. Harper [16] studied morphisms for resistive electrical networks to solve Kirchhoff's problem in 2014. Alman, Lian and Tran [1] in 2015 found a new result on circular planner electrical network.

3. FROM CIRCUIT TO GRAPH:

A graph can be obtained from an electrical circuit. We identify the graph G= (V, E) where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by {i,j} ignoring the direction. Similarly the notation (i,j) can be used for oriented edges, where i is the start vertex and j is the end vertex. In general current source and voltage source are replaced by open circuit and short circuit respectively.

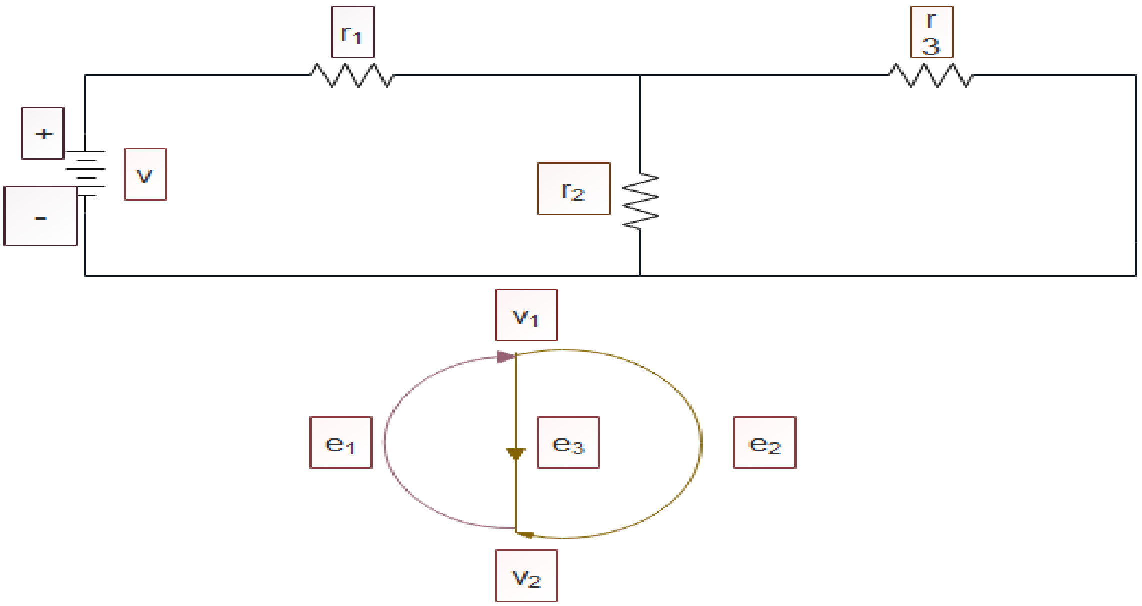

Figure 1: A circuit and its corresponding graph

3.1 Matrices Associated to a Graph:

a) Fundamental Tie set matrix (Fundamental Loop Matrix) [B] :

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here, all loops obtained from a particular tree forms the rows and all branches form the columns such that

$$b_{ij} = \begin{cases} 1, \text{ if branch } b_j \text{ in loop } i \text{ are in the same direction} \\ -1, \text{ if branch } b_j \text{ in loop } i \text{ are in the opposite direction} \\ 0, \text{ if branch } b_j \text{ is not in loop } i \end{cases}$$

b) Branch Impedance Matrix [Z_b]:

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entries are zero.

3.2 Application of Graph theory in Network Equilibrium Equations:

If V_{SK} be the voltage source in a branch k having impedance z_k and carrying current i_k, then the branch voltage $v_k = z_k i_k + V_{SK}$

If we consider this equation for the whole circuit then in matrix form this can be written as $[V_b] = [Z_b][I_b] + [V_S]$ where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_S]$ is the column vector of source voltage. According to Kirchhoff's Voltage law the algebraic sum of voltages in any closed path of network traversed in a single direction is zero. In matrix form it is represented as

$$[B][V_b] = 0$$

$$\Rightarrow [B]([Z_b][I_b] + [V_S]) = 0$$

$$\Rightarrow [B][Z_b][I_b] = -[B][V_S]$$

Also the branch current matrix equation is $[I_b] = [B^T][I_L]$, where $[I_L]$ is the loop current matrix and $[B^T]$ is the transpose of [B].

So, we get $$[B][Z_b][B^T][I_L] = -[B][V_S]$$

We will use this equation to analyze the following circuits and also in the equivalent circuits obtained by Thevenin's theorem.

4. Finding Current using Graph Theoretic Approach:

Consider the circuit with one unknown resistance $'r_i'$, $i = 1, 2, 3, 4$ and unknown voltage 'v', all are non zero.

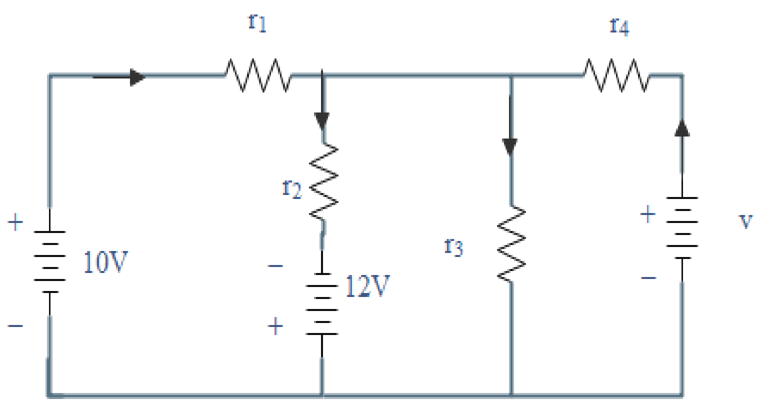

Figure 2: Circuit with unknown resistance and voltage

We will find the current through resistance $'r_3'$ by Graph theoretic approach and verify it with Thevenin's theorem.

4.1 Graph Theoretic Approach:

The graph of the main circuit is

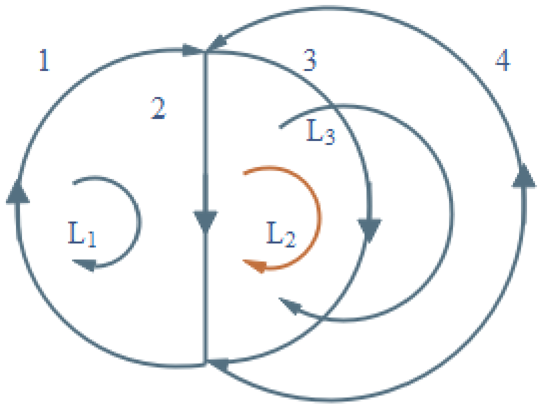

Figure 3: Graph of the circuit

From the graph we obtain the following matrices:

$$B=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & -1 & 0 & -1\end{bmatrix},\quad Z_b=\begin{bmatrix}r_1 & 0 & 0 & 0\\ 0 & r_2 & 0 & 0\\ 0 & 0 & r_3 & 0\\ 0 & 0 & 0 & r_4\end{bmatrix},$$

$$B^T=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & -1\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix},\quad I_L=\begin{bmatrix}I_{L_1}\\ I_{L_2}\\ I_{L_3}\end{bmatrix},\quad V_s=\begin{bmatrix}10\\ 12\\ 0\\ v\end{bmatrix}$$

So putting in

$$[B][Z_b][B^T][I_L]=-[B][V_s]$$

$$\Rightarrow\begin{bmatrix}r_1+r_2 & -r_2 & -r_2\\ -r_2 & r_2+r_3 & r_2\\ -r_2 & r_2 & r_2+r_4\end{bmatrix}\begin{bmatrix}I_{L_1}\\ I_{L_2}\\ I_{L_3}\end{bmatrix}=\begin{bmatrix}-22\\ 12\\ v+12\end{bmatrix}$$

$$\Rightarrow X\ I_L=Y$$

So there exist a unique circuit for unique values of ‘r_i’ and ‘v’.

Solving we have

$$I_{L_1}=\frac{-10r_2r_3-10r_2r_4-22r_3r_4+r_2r_3v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4}A(\text{Ampere}),\ I_{L_2}=\frac{12r_1r_4-10r_2r_4-r_1r_2v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4}A,$$

$$I_{L_3}=\frac{12r_1r_3-10r_2r_3+r_1r_2v+r_1r_3v+r_2r_3v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4}A\ \text{where}\ r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4\neq 0$$

Again from

$$[I_b]=\left[B^T\right][I_L]$$

$$\Rightarrow\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}=\begin{bmatrix} \dfrac{-10r_2r_3-10r_2r_4-22r_3r_4+r_2r_3v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4} \\ \dfrac{-12r_1r_3-12r_1r_4-22r_3r_4-r_1r_3v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4} \\ \dfrac{12r_1r_4-10r_2r_4-r_1r_2v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4} \\ \dfrac{-12r_1r_3+10r_2r_3-r_1r_2v-r_1r_3v-r_2r_3v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4} \end{bmatrix}$$

Here $'i_k'$ is the kth branch current.

So we see that current through resistor $'r_3'$, $i_{r_3}=i_3=\dfrac{12r_1r_4-10r_2r_4-r_1r_2v}{r_1r_2r_3+r_1r_2r_4+r_1r_3r_4+r_2r_3r_4}$

4.2 Verification by Thevenin's Theorem:

Thevenin's Theorem: Any two terminal bilateral linear d.c.circuits can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

To construct Thevenin's equivalent circuit first we remove the resistor 'r_3' from the main circuit and considering the equivalent voltage along the connected nodes as V_{oc}

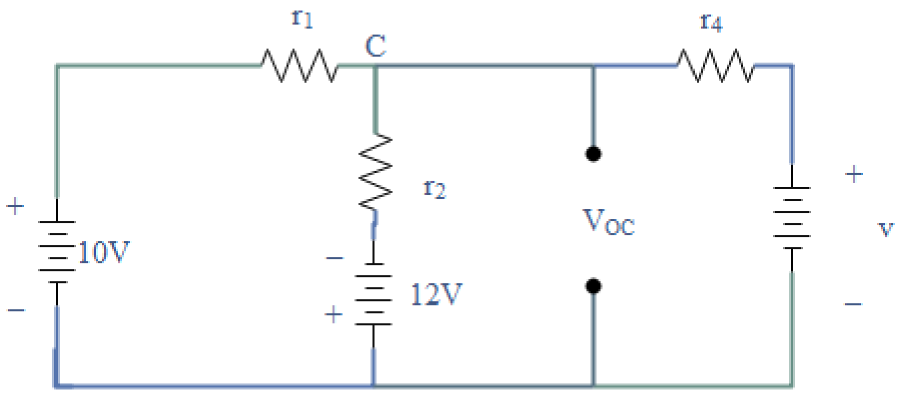

Figure 4: Circuit to find V_{oc}

Now applying Kirchhoff's law at node C

$$\frac{V_{oc}-10}{r_1}+\frac{V_{oc}+12}{r_2}+\frac{V_{oc}-v}{r_4}=0$$

$$\Rightarrow V_{oc}=\frac{10r_2r_4+r_1r_2v-12r_1r_4}{r_2r_4+r_1r_4+r_1r_2}V\ (Volt)$$

Now to find the equivalent resistance $R_{Th,}$ removing the voltage sources

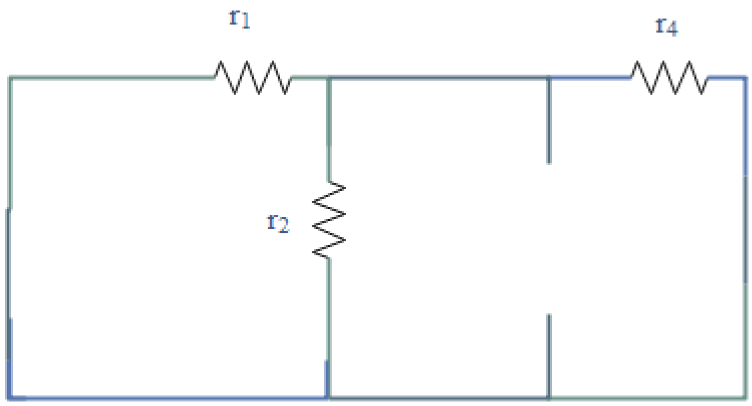

Figure 5: To find equivalent resistance

Since r_1, r_2, r_4 are in parallel so

$$\frac{1}{R_{Th}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_4}$$

$$\Rightarrow R_{Th} = \frac{r_1 r_2 r_4}{r_2 r_4 + r_1 r_4 + r_1 r_2} \Omega$$

Hence Thevenin's equivalent circuit is given by

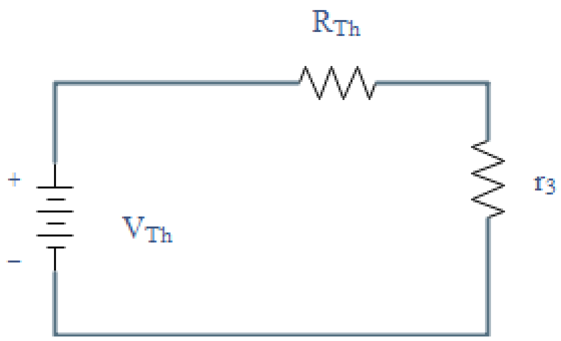

Figure 6: Thevenin's equivalent circuit

So the current through resistor $'r_3'$,

$$i_{r_3} = \frac{V_{Th}}{R_{Th} + r_3} = \frac{\dfrac{10 r_2 r_4 + r_1 r_2 v - 12 r_1 r_4}{r_2 r_4 + r_1 r_4 + r_1 r_2}}{\dfrac{r_1 r_2 r_4}{r_2 r_4 + r_1 r_4 + r_1 r_2} + r_3} = \frac{10 r_2 r_4 + r_1 r_2 v - 12 r_1 r_4}{r_1 r_2 r_4 + r_2 r_3 r_4 + r_1 r_3 r_4 + r_1 r_2 r_3} \Omega$$

So we get the same current as in matrix method.
In case of matrix method negative sign is just showing that current is in opposite direction.

5. Results: In $X\, I_L = Y$ we see that

1) The matrix X is symmetric.
2) The diagonal elements of X

$a_{11} = r_1 + r_2$ = sum of resistances in L_1

$a_{22} = r_2 + r_3$ = sum of resistances in L_2

$a_{33} = r_2 + r_4$ = sum of resistances in L_3

So the a_{ii} element (diagonal) of matrix X is the sum of the resistances of loop L_i.

3) Without Using nodal analysis and mesh analysis we can find the loop current and branch current by using graph theory only and putting the values in the obtained formulae.

6. CONCLUSION:

From the above study we observed that the result obtained by Thevenin's Theorem is same as the result obtained by Graph theoretic approach using Network equilibrium equation in a circuit with unknown resistances and unknown voltage. So the graph theoretic approach is verified by existing Thevenin's Theorem. The value of currents can be obtained for different value of resistances and voltages by network equilibrium equation without knowing the nodal and mesh analysis of electrical circuits. When an electrical circuit contains many branches then it is laborious to use Thevenin's theorem to find branch current and loop currents etc. This difficulty can be solved with the help of matrix method because in this method we can multiply n^{th} order matrices which can be obtained from circuits containing n loops.

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