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THE MATRIX OF A (3, 2) JECTION OPERATOR

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ABSTRACT : In the present paper, our objective is to find a matrix corresponding to a given (3, 2)-jection operator in order that when we think of the matrix as an object that acts on a vector by multiplication to produce the same (3,2)-jection operator.

Index terms :Matrix, Linear span, Linear operator, Base, (3, 2)-jection operator.

I. INTRODUCTION :

We know that each linear transformation from an n-dimensional vector space U(F) to an m-dimensional vector space V(F) corresponds to an $m \times n$ matrix over field F, which depends upon the bases of vector spaces U and V.

This paper presents an organized procedure of constructing matrix representation for a (3, 2)-jection operator which is a suitable generalization of projection.

Even though the readers are expected to know the various results connected with the linear algebra and functional analysis yet for the sake of convenience and ready reference we shall deal in a nutshell the various definitions and concepts which will be of great help to readers in the study of the facts of this paper.

II. IMPORTANT DEFINITIONS :

(1) **Matrix :**A matrix over a field F or simply, a matrix A (when F is implicit) is a rectangular arrangement of scalars.

(2) **Basis :**A non-void subset B of a vector space V is said to be a basis for V if

(a) B spans V and

(b) B is linearly independent set of vectors in V.

(3) **Linear spans :** The set of all linear combinations of a set of vectors is called the span of that set of vectors.

(4) **Linear operator :** A linear transformation from a vector space to itself, is called a linear operator.

(5) **(3, 2)-jection operator :** A linear operator E on a linear space V such that $E^2 = E^2$, is called a (3, 2)-jection operator.

III. METHODOLOGY :

Here we write $z = (x,y) \in R^2$, as a column vector $\begin{bmatrix} x \\ y \end{bmatrix}$.

Since $z = I_2 z$

$$= [e_1\ e_2] \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{where } e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= xe_1 + ye_2$$

Let E be a (3,2)- jection on R^2 then

$$E(z) = E(xe_1 + ye_2)$$

$$= xE(e_1) + yE(e_2)$$

$$= [E(e_1)\ \ E(e_2)] \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= Az \qquad \text{where } A = [E(e_1)\ \ E(e_2)]$$

Hence, we state that

If E be a (3,2)-jection on R^2 then there exists a unique matrix A such that $E(z) = Az \ \forall\ z \in R^2$. The matrix A is called the standard matrix for E.

IV. MAIN RESULT :

The results are the following :

[1] Matrix representation of E(x, y) = (0, cx) with respect to the ordered basis $\{e_1, e_2\}$ where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

When z = (x, y) and E(z) be written as column vectors then $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} 0 \\ cx \end{bmatrix}$

Now,

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ c \end{bmatrix}$$

And $$E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the standard matrix of E is $A = \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 & + & 0 \\ cx & + & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ cx \end{bmatrix}$$

$$= E(z)$$

[2] Matrix representation of E(x, y) = (by, 0)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

When z = (x, y) and E(z) be written as column vectors then $z = \begin{bmatrix} x \\ y \end{bmatrix}$ & $E(z) = \begin{bmatrix} by \\ 0 \end{bmatrix}$

We have

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And $\quad E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$

So, the standard matrix of E is $A = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 & + & by \\ 0 & + & 0 \end{bmatrix}$$

$$= \begin{bmatrix} by \\ 0 \end{bmatrix}$$

$$= E(z)$$

[3] Matrix representation of $E(x, y) = (by, y)$ with respect to the ordered basis $\{e_1, e_2\}$ where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

When z = (x, y) and E(z) be written as column vectors then $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} by \\ y \end{bmatrix}$

Now

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ 1 \end{bmatrix}$

So, the standard matrix of E is $A = \begin{bmatrix} 0 & b \\ 0 & 1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 0 & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 + by \\ 0 + y \end{bmatrix}$$

$$= \begin{bmatrix} by \\ y \end{bmatrix}$$

$= E(z)$

[4] Matrix representation of $E(x, y) = (0, y)$ with respect to the ordered basis $\{e_1, e_2\}$ where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

We write z = (x, y) and E(z) as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} 0 \\ y \end{bmatrix}$

Then,

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 & + & 0 \\ 0 & + & y \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ y \end{bmatrix}$$

$$= E(z)$$

[5] Matrix representation of E(x, y) = (0, cx + y)with respect to the ordered basis{e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

We write z = (x, y) and E(z) as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} 0 \\ cx + y \end{bmatrix}$

Then,

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ c \end{bmatrix}$$

And $\quad E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 0 & 0 \\ c & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} 0 & + & 0 \\ cx & + & y \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ cx + y \end{bmatrix}$$

$$= E(z)$$

[6] Matrix representation of E(x, y) = (x, 0)with respect to the ordered basis {e_1, e_2} where $\mathbf{e_1} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}, \mathbf{e_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}$.

Interpretation :

Let us write z = (x, y) and E(z) by the column vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ and $\begin{bmatrix} x \\ 0 \end{bmatrix}$ respectively.

Then,

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And $$E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So, the standard matrix of E is $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x & + & 0 \\ 0 & + & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ 0 \end{bmatrix}$$

$$= E(z)$$

[7] Matrix representation of E(x, y) = (x, y)with respect to the ordered basis {e_1, e_2} where $\mathbf{e_1} = \begin{bmatrix} \mathbf{1} \\ \mathbf{0} \end{bmatrix}, \mathbf{e_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}$.

Interpretation :

If z = (x, y) and E(z) be written as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} x \\ y \end{bmatrix}$

Now

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Verification :

$$Az = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$= \begin{bmatrix}x & + & 0\\0 & + & y\end{bmatrix}$$

$$= \begin{bmatrix}x\\y\end{bmatrix}$$

$$= E(z)$$

[8] Matrix representation of E(x, y) = (x, cx)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) be written as column vectors so that $z = \begin{bmatrix}x\\y\end{bmatrix}, E(z) = \begin{bmatrix}x\\cx\end{bmatrix}$

Now,

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\c\end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

So, the standard matrix of E is $A = \begin{bmatrix}1 & 0\\c & 0\end{bmatrix}$

Verification :

$$Az = \begin{bmatrix}1 & 0\\c & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$= \begin{bmatrix}x & + & 0\\cx & + & 0\end{bmatrix}$$

$$= \begin{bmatrix}x\\cx\end{bmatrix}$$

$= E(z)$

[9] Matrix representation of E(x, y) = (x + by, 0)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (x + by, 0) be written as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} x+by \\ 0 \end{bmatrix}$

We have

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

And $$E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

Hence, the standard matrix of E is $A = \begin{bmatrix} 1 & b \\ 0 & 0 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 1 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x+by \\ 0 \end{bmatrix}$$

$$= E(z)$$

[10] Matrix representation of E(x, y) = $\left(x+by, -\frac{1}{b}x - y\right)$ with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = $\left(x+by, -\frac{1}{b}x - y\right)$ be written as column vectors such that

$$z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} x+by \\ -\frac{1}{b}x - y \end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1 + 0\\ -\frac{1}{b} - 0\end{bmatrix} = \begin{bmatrix}1\\ -\frac{1}{b}\end{bmatrix}$$

And $$E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0 + b\\ 0 - 1\end{bmatrix} = \begin{bmatrix}b\\ -1\end{bmatrix}$$

Hence, the standard matrix of E is $A = \begin{bmatrix}1 & b\\ -\frac{1}{b} & -1\end{bmatrix}$

Verification :

$$Az = \begin{bmatrix}1 & b\\ -\frac{1}{b} & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$= \begin{bmatrix}x + by\\ -\frac{1}{b}x - y\end{bmatrix}$$

$$= E(z)$$

[11] Matrix representation of E(x, y) = $\left(-x + by, -\frac{1}{b}x + y\right)$ with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = $\left(-x + by, -\frac{1}{b}x + y\right)$ be written as column vectors such that

$$z = \begin{bmatrix}x\\y\end{bmatrix}, E(z) = \begin{bmatrix}-x + by\\ -\frac{1}{b}x + y\end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1 + 0\\ -\frac{1}{b} + 0\end{bmatrix} = \begin{bmatrix}-1\\ -\frac{1}{b}\end{bmatrix}$$

And $$E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0 + b\\ 0 + 1\end{bmatrix} = \begin{bmatrix}b\\ 1\end{bmatrix}$$

Hence, the standard matrix of E is $A = \begin{bmatrix} -1 & b \\ -\frac{1}{b} & 1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} -1 & b \\ -\frac{1}{b} & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -x+by \\ -\frac{1}{b}x+y \end{bmatrix}$$

$$= E(z)$$

[12] Matrix representation of E(x, y) = (–x + y, –x + y)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (–x+y, –x + y) be written as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} -x+y \\ -x+y \end{bmatrix}$

We have

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+0 \\ -1+0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0+1 \\ 0+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} -x+y \\ -x+y \end{bmatrix} = E(z)$$

[13] Matrix representation of E(x, y) = (–x–y, x + y)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (–x–y, x + y) be written as column vectors such that

$$z=\begin{bmatrix}x\\y\end{bmatrix}, E(z)=\begin{bmatrix}-x-y\\x+y\end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-1\\1\end{bmatrix}$

Hence, the standard matrix of E is $A=\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}$

Verification :

$$Az=\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$=\begin{bmatrix}-x-y\\x+y\end{bmatrix} = E(z)$$

[14] Matrix representation of E(x, y) = $\left(\frac{x+y}{2},\frac{x+y}{2}\right)$ with respect to the ordered basis {e_1, e_2} where $e_1=\begin{bmatrix}1\\0\end{bmatrix}$, $e_2=\begin{bmatrix}0\\1\end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = $\left(\frac{x+y}{2},\frac{x+y}{2}\right)$ be written as column vectors such that

$$z=\begin{bmatrix}x\\y\end{bmatrix}, E(z)=\begin{bmatrix}\frac{x+y}{2}\\\frac{x+y}{2}\end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\\\frac{1}{2}\end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & \frac{1}{2}\end{bmatrix}$

Verification :

$$Az = \begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$= \begin{bmatrix}\frac{x+y}{2}\\\frac{x+y}{2}\end{bmatrix}$$

$$= E(z)+$$

[15] Matrix representation of E(x, y) = (kx + ky, –kx – ky)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix}1\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (kx + ky, –kx – ky) be written as column vectors such that $z = \begin{bmatrix}x\\y\end{bmatrix}, E(z) = \begin{bmatrix}kx+ky\\-kx-ky\end{bmatrix}$

We have

$$E(e_1) = E\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}k\\-k\end{bmatrix}$$

And $E(e_2) = E\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}k\\-k\end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix}k & k\\-k & -k\end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} k & k \\ -k & -k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} kx + ky \\ -kx - ky \end{bmatrix}$$

$$= E(z)$$

[16] Matrix representation of E(x, y) = (λ x + λ y, (1−λ)x + (1−λ)y)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (λ x + λ y, (1−λ)x + (1−λ)y) be written as column vectors such that $z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} \lambda x + \lambda y \\ (1-\lambda)x + (1-\lambda)y \end{bmatrix}$

We have

$$E(e_1) = E \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ 1-\lambda \end{bmatrix}$$

And $E(e_2) = E \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda \\ 1-\lambda \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} \lambda & \lambda \\ 1-\lambda & 1-\lambda \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} \lambda & \lambda \\ 1-\lambda & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \lambda x + \lambda y \\ (1-\lambda)x + (1-\lambda)y \end{bmatrix}$$

$$= E(z)$$

[17] Matrix representation of E(x, y) = $\left(\frac{x-y}{2}, \frac{-x+y}{2}\right)$ with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = $\left(\frac{x-y}{2}, \frac{-x+y}{2}\right)$ be written as column vectors such that

$$z = \begin{bmatrix} x \\ y \end{bmatrix}, E(z) = \begin{bmatrix} \frac{x-y}{2} \\ \frac{-x+y}{2} \end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

And $\quad E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

[18] Matrix representation of E(x, y) = (x – y, x – y)with respect to the ordered basis {e_1, e_2} where $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Interpretation :

Here z = (x, y) and E(z) = (x – y, x – y) be written as column vectors such that

$$z = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } E(z) = \begin{bmatrix} x-y \\ x-y \end{bmatrix}$$

We have

$$E(e_1) = E\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

And $\quad E(e_2) = E\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

Hence, the standard matrix of E is $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

Verification :

$$Az = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x-y \\ x-y \end{bmatrix}$$

$$= E(z)$$

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