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FUZZY UNIFORMITIES ON FUNCTION SPACE

Dr.EKTA TRIPATHI
Ph. D. in Mathematics
Email ID – ektatripathi08@gmail.com

ABSTRACT:-

In this paper we study several uniformities on a function space and show that the fuzzy topology associated with the fuzzy uniformity of uniform convergence is jointly fuzzy continuous on $cf(X,Y)$, the collection of all fuzzy continuous function from a fuzzy topology space X into a fuzzy inform space Y. We define fuzzy uniformity of uniform convergence on starpluscompacta and show that its corresponding fuzzy topology is the starplus compact open fuzzy topology. Moreover, we introduce the notion of fuzzy equicontinuity and fuzzy uniform equicontinuity on fuzzy subset of a function space and study their properties. We also elaborate on the pointwise fuzzy uniformity, fuzzy uniformity of uniform and fuzzy uniformity of uniform convergence of starpluscompacta and their associated fuzzy topology on function space. The fuzzy topology of uniform convergence on starpluscompacta is the starplus compact open fuzzy topology.

INTRODUCTION:-

The notion of uniform space was first time given by Andre Weil in 1937. But the first systematic exposition of the theory of uniform space was given by Bourbaki in 1940. Weil elaborate the topology associated with a uniformity and proved that a topological space is uniformizable if it is completely regular. He extended the notion of uniform continuity and uniform isomorphism to the framework of uniform spaces and obtained the uniform space version of AlezandreffUryshonmetrization theorem. The concept of function space was evolved around the close of the nineteenth century and the study of function spaces began with the work of Ascoli, Arzela and Hadamard. The uniformity of pointwise convergence and uniform convergence were first defined and studied by FOX .

The study of topologies on function space is an active area of research. The study of useful fuzzy topology and uniformities on function spaces besides their intrinsic, interest is important from applications viewpoint. The first effort in this direction was made by Peng in 1984. Burton defined analogues of the uniformities of pointwise convergence and uniform convergence. We defined three different fuzzy topologies on function space, which are analogues of the topology of joint continuity in general topology.

In this paper we elaborate on the pointwise fuzzy uniformity, fuzzy uniformity of uniform convergence and fuzzy uniformity of uniform convergence on starpluscompacta and their associated fuzzy topologies on a function space.

In this paper, the closed unit interval [0,1] will be denoted by I. The symbols I_0 and I_1 will stands for the intervals (0,1] and [0,1) respectively.

Defination:-

For a fuzzy set μ in X, the set $\mu^{\propto} = \{x \in X : \mu(x) > \propto\}$ and $\mu_{\propto} = \{x \in X : \mu(x) \geq \propto\}$ are called the strong α – level set of μ and the weak α – set of μ, respectively. The set $\{x \in X : \mu(x) > 0\}$ is called the support of μ and denoted by supp μ.

Defination:-

A fuzzy set μ in a fts$(X, \mathfrak{I})$is said to be starplus compact if $\mu^{\propto}$ is compact in $(X, i_{\propto}(\mathfrak{I})$ for each $\propto \in I_1$. The ft$(X, \mathfrak{I})$sis said to be starplus compact if $(X, i_{\propto}(\mathfrak{I})$ is compact for each $\propto \in I_1$.

Let X be a non – empty set and $(X, \mathfrak{I})$ be a fuzzy topological space. Let Y^x denote the collection of all functions from X into Y and let $\mathfrak{I}$be a non-empty subset of Y^x

Defination:-

For each $x \in X$, let the map $e_x : \mathfrak{I} \to y$ be defined by $e_x(f) = f(x)$. We called the pointwise fuzzy topology on $\mathfrak{I}$ and is denoted by T_p. The pair $(\mathfrak{I}, T_p)$ is called the pointwise fuzzy function space.

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FUZZY UNIFORMITIES ON FUNCTION SPACE:-

Let x be a non-empty set and (Y, u) be a fuzzy uniform space. Let Y^x denote the collection of maps from X into Y. let $\mathfrak{J}$ be a non-empty subset of Y^x. The fuzzy topology associated with the fuzzy uniformity of uniform convergence is jointly fuzzy continuous on $C_f(x, y)$.

Defination:-

The initial fuzzy uniformity $\mu_p$on $\mathfrak{J}$ generated by the collection of maps $(C_x : x \in X)$ is called the pointwise convergence on $\mathfrak{J}$. The pair $(\mathfrak{J}, \mu_p)$ is called the pointwise fuzzy uniform space.

Theorem:-

Let X be a non-empty set and (Y, μ) be a uniform space. Let μ_p denote the pointwise uniformity on $\mathfrak{J}$ and μ_p be the pintwise fuzzy uniformity on $\mathfrak{J}$, where Y is endowed with the fuzzy uniformity $w_\mu(\mu)$. Then $w_\mu(\mu_p) = \mu_p$.

Proof:-

Let $\mu_p \wedge_{i=1}^{n} (e_x \times e_x)^{-1} (V_i)$ be a basic element for the fuzzy uniformithμ_p, where Y is endowed with the fuzzy uniformity $w_\mu(\mu)$. Since $V_i \in w_\mu(\mu), U\ for\ each\ \alpha \in I$ and hence, $[\cap_{i=1}^{n} (e_x \times e_x)^{-1} (V_i)] \in \mu_p$. This shows that $[\wedge_{i=1}^{n} (e_x \times e_x)^{-1} (V_i)]^\alpha \in \mu_p$.

Thus we have, $w_\mu(\mu_p) \supseteq \mu_p$

The proof of the opposite inclusion, $w_\mu(\mu_p) \subseteq \mu_p$ is similar to the one given above.

Proposition:-

Let X be a set and (Y,μ) be a fuzzy uniform space. Let μ_p be the fuzzy uniformity of pointwise convergence on $\mathfrak{J}$. Then the each $\alpha \in I_p$ the $\alpha-$level uniformity $i_{\mu\alpha}(U_p)$ is the uniformity of pointwise convergence on $\mathfrak{J}$ with respect to $i_{\mu,\alpha}(U)$ on Y.

Proof:-

Let $(e_x \times e_x)^{-1}(V)$ be a subbasic element in U_p.

Then for each $\alpha \in I_1$,

$$[(e_x \times e_x)^{-1}(V)]^{\beta} = \{(f,g) \in \mathfrak{J} \times \mathfrak{J} : v(f(x), g(x)) > \beta\}, \beta \in [0,1-\alpha)$$

$$= \{(f,g) \in \mathfrak{J} \times \mathfrak{J} : (f(x), g(x)) \in V^{\beta}\}$$

$$=(e_x \times e_x)^{-1}(V^{\beta})$$

Which is a subbasic element in the pointwise uniformity on ζ, where γ is endowed with the uniformity $i_{\mu,\alpha}(U)$

FUZZY UNIFORMITY ON UNIFORM CONVERGENCE:-

The notion of fuzzy uniformity of uniform convergence on a function space $\mathfrak{J} \subset Y^X$ (Which was initiated by Burton) Where X is a nonempty set and (Y,U) is a fuzzy uniform space.

Definition:-

For each $V \in U$, the fuzzy set $W_V = \wedge x \in X\, (e_x \times e_x)^{-1}(V)$ in $\mathfrak{J} \times \mathfrak{J}$ Where $e_x : \mathfrak{J} \to Y$ is the evaluation map, is defined by, $W_V(f,g) = \wedge x \in X\, V(f(x), g(x))$

Let β be the collection of all W_V, where V varies over U.

Proposition:-

The collection β is a base for a fuzzy uniformity on $\mathfrak{J}$.

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Example:-

If $\mu \in I^{\mathfrak{I}}$, then the section of W_V over μ is defined by

$$W_V < \mu > (f) = \bigwedge_{x \epsilon X} V < e_x(\mu) > (f(x))$$

$$= \inf_{x \epsilon X} \{ \sup_{g \epsilon \mathfrak{I}} (e_x(\mu)(g(x)) \wedge V(g(x)), f(x)) \}$$

For each $f \in \mathfrak{I}$.

The following terminology and notions of a fuzzy uniformity defined by lower.

Definition:-

A fuzzy uniformity on X is a subset $\cup \subset I^{X \times X}$ which satisfies the following canditions:

i) U is a pre filter.

ii) $\widehat{U} = U$, i.e. for every family $(V_{\in})_{\in \in I_0} \in U^I$

$$\Rightarrow \sup_{\vee \in I_0} (V_{\epsilon} - \in) \in \cup$$

iii) For all $V \in \cup$, and for all $x \in X$, $V(x, x) = 1$

iv) For all $V \in \cup$, $S^V \in \cup$,

v) For all $V \in \cup$, and for all $\in^{\in I_0}$ there exists $V_{\in} \in \cup$ such that $V_{\in}\ 0\ V_{\in} - \in \leq V$.

The pair (X,U) is called a fuzzy uniform space.

Definition:-

A subset $\beta \subset I^{X \times X}$ is called a base for a fuzzy uniformity if and only if the following conditions hold

i) β is a prefilter basis.

ii) For all $\beta \in \beta$ and for all $x \in X, \beta(x, x) = 1$

iii) For all $\beta \in \beta$ and for all $\in^{\in I_o}$, there exists $\beta_{\in} \in \beta$ such that $\beta_{\in} - \in \leq s^{\beta}$

iv) For all $\beta \in \beta$ and for all $\in^{\in I_o}$, there exists $\beta_{\in} \in \beta$ such that $\beta_{\in} 0\ \beta_{\in} - \in \leq \beta$

Definition:-

The fuzzy uniformity U_u generated by β is called the fuzzy uniformity of uniform convergence. The fuzzy topology associated with U_u is called the fuzzy topology on uniform convergence and it is denoted by T_u.

In the following results we give a short description of the concepts that Burton uses relative to the fuzzy uniform convergence.

Theorem:-

Let F be a prefilter in $Y^X$with $c(F) = \bar{c}(F)$. Then F is U_u −cauchy, $\alpha \leq \bar{c}$, $\lim_{Up}(F)(f) \geq \alpha \Rightarrow \lim_{Up}(F)(f) \geq \alpha$.

Theorem:-

If (X,U) is complete then (Y^X, U_u) is complete.

Theorem:-

If it is closed fuzzy set in (Y^X, U_P) and for all $x \in X, e_x(\mu)$ is complete in (X,U) then the fuzzy set μ is complete in (Y^X, U_u).

Proposition:-

If $W_{vi}, 1 \leq i \leq n$ are members of U_u, then the following hold:

i) $\wedge_{i=1}^{n} W_{vi} = W_{(\wedge_{i=1}^{n})}$ and

ii) $W_V^{\propto} = W_{V^{\propto}}, for \ \propto \in I$

Proof:-

i) For each i

$$W_P = \bigwedge_{x \in X} (e_x \times e_x)^{-1} \ (vi)$$

$$\begin{aligned} \wedge_{i=1}^{n} W_P &= \wedge_{i=1}^{n}\{\wedge_{x\in X}^{n}(e_x \times e_x)^{-1}(vi)\} \\ &= \wedge_{x\in X}(e_x \times e_x)^{-1}(\wedge_{i=1}^{n} Vi) \\ &= W_{(\wedge_{i=1}^{n} Vi)} \end{aligned}$$

ii) For each $\alpha \in I_1$

$$\begin{aligned} W_{V^\alpha} &= \cap_{x\in X}\{(f,g):(f(x),g(x)) \in V^\alpha\} \\ &= \cap_{x\in X}\{(f,g):(e_x \times e_x)(f,g) \in V^\alpha\} \\ &= \cap_{x\in X}\{(f,g):(f,g) \in (e_x \times e_x)^{-1}(V^\alpha)\} \\ &= \cap_{x\in X}\{(f,g):(f,g) \in [(e_x \times e_x)^{-1}(V)]^\alpha\} \\ &= \cap_{x\in X}[(e_x \times e_x)^{-1}(V)]^\alpha \\ &= [\wedge_{x\in X}(e_x \times e_x)^{-1}(v)]^\alpha \\ &= (W_v)^\alpha \end{aligned}$$

Fuzzy uniformity of uniform convergence on starpluscompacta.

Definition:-

Let X be a fuzzy topological space and let (Y,U) be a fuzzy uniform space. Let S be the collection of all starplus compact fuzzy sets in X.

For each $k \in S$ and $v \in U$, define a fuzzy set $W(k,v)$. $\mathfrak{I} \times \mathfrak{I} \to I$by

$$W(k,v)(f,g) = \wedge_{x\in sup\ pk}(e_x \times e_x)^{-1}(V)(f,g)$$

$$= \wedge_{x\in sup\ pk} V(f(x),g(x))$$

Then the collection of all fuzzy sets $\{W(k,v): k \in S, V \in U\}$ is a base for a fuzzy uniformity on $\mathfrak{I}$ and is called the **fuzzy uniformity of uniform convergence on starpluscompacta**.

Theorem:-

Let $\Im$ be the set of all fuzzy continuous maps from a topologically generated fts(X,T_X) into a fuzzy uniform space (Y,U), Then the fuzzy topology of fuzzy uniform convergence on starpluscompacta is the starplus compact open fuzzy topology.

Proof:-

Let $\cup_{*C}^{+}$ be the fuzzy uniformity of uniform convergence on starpluscompacta. Since X is a topologically generated fts, a subset K of X is compact in (X, $i_o(T_X)$) if and only if XK is starplus compact in (X,T_X), Hence $i_{u\alpha}(\cup_{*C}^{+}) = u_{UC}$, where u_{UC} denotes the uniformity of uniform convergence on compaca, where X is endowed with the topology$i_o(T_X)$ and Y is equipped with the uniformity $i_{u\alpha}(U)$.

So, by theorem, $T_1(i_{u\alpha}(\cup_{*C}^{+}))$ is the compact open topology for each $\alpha \in I_1$

Again by theorem $T_1\big(i_{u\alpha}(\cup_{*C}^{+})\big) = i_\alpha(T_{*C}^{+})$ for each $\alpha \in I_1$ and $i_\alpha\big(T(\cup_{*C}^{+})\big) = T\big(i_{u\alpha}(\cup_{*C}^{+})\big) = i_\alpha(T_{*C}^{+})$for each $\alpha \in I_1$.

This completes the proof.

REFERENCES:-

i) R. H. Fox, On topologies for function spaces, Bull, Amer. Math, Soc. 51(1945) 429-432.
ii) I. W. Alderton, Function spaces in fuzzy topology, fuzzy sets and systems, 32(1989), 115-124.
iii) G. Juger, Fuzzy uniform convergence and equicontinuity, Fuzzy Sets and systems 109(2000), 187-198.
iv) J. L. Kelley, General topology, Van nosterand, New York 1955.
v) J. K. Kohli and A.R. Prasannan, Fuzzy topologies on function spaces, fuzzys sets and systems 116(3) (2000), 415-420.
vi) R.Arens and J. Dugundiji, "topologies for function spaces, Pacific J. Math, 1(1951) PP 5-31