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ANALYTIC FORMULATION ON VIBRATION OF ORTHOTROPIC CLAMPED PLATE UNDER IN-PLANE FORCES

Ahmed M. Farag El Sheikh*

ABSTRACT

This paper is concerned with eigen values on buckling and vibration of orthotropic rectangular clamped plate. The present study attempts to achieve closed form solutions for orthotropic clamped plates resting on elastic base, in a novel way based on wide panel-transition matrix technique. Strip technique is employed with transition matrix method to develop the analytical solutions in series forms. Increasing the accuracy of the transition matrix is the main idea to reduce the number of strips of the decomposed plate domain. The buckling and natural frequency parameters of clamped plate are investigated under the effect of the un-axial and biaxial in-plane forces. Analytic results of vibration natural frequencies are obtained for orthotropic clamped plate under in-plane forces. Also the effects of the aspect ratios and coefficients of elastic foundation on the behavior of rectangular plates are discussed. The obtained analytical results may serve as benchmark solutions for such plates. Numerical results are obtained and discussed for a wide range of orthotropic properties and foundation coefficients. The validity of the present method is examined by means of several numerical examples compared with those available in the published papers. The obtained results proved accuracy and validity of the achieved technique.

KEYWORDS:

Closed Form;
Transition Matrix;
Buckling;
Clamped Plate;
Wide Panel

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1. INTRODUCTION

Compressed plates are often existed when the target of stiffness enhancing and designated strengthening is purposed. Kukla and Skalmiersk gave a brief historical review recently on this subject [1]. Thought an exact solution, Xiaing an Wang. [2] investigated buckling and vibration for plates with two opposite edges simply supported in the presence of in-plane forces. Xiaing and Wei [3] extended this exact solution to solve buckling and vibration of stepped rectangle Mindlin plates. Vescovini et al [4] applied the Ritz method to analyze free vibration and buckling of anisotropic plates under classical boundary conditions. Lopatin and E.V. Morozov [5] applied a non-uniform in-plane compressive force to check the buckling of orthotropic plate via the Kantorovich technique and Galerkin approach. An analytical approach for buckling of simply supported sandwich plates has been achieved by Kheirikhah et al [6] using Navier's solution. Static, free vibration, and buckling analysis of laminated composite Reissner-Mindlin plates were studied by Chien H. Thai1et al [7] using NURBS-based isogeometric approach. Xinwei Wang et al [8] applied differential quadrature to analyze the buckling of plates under cosine- compressive forces. A Reddy type, third order shear deformation theory of plates has been applied to the development of two versions of finite strip method (FSM), namely semi-analytical and spline methods, to predict the behavior of the moderately thick plates containing cutout. Fazzolari et al [9] achieved an exact stiffness element for free vibration study of composite plates. Mei and Yang [10] investigated the free vibrations of finite element plates subjected to complex middle-plane force systems. Xiang Liua et al [11] offered highly accurate analytical solutions for buckling and wrinkling study of orthotropic plates via Fourier series.

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A semi-analytical method for vibration of elastically restrained thick graded plates on elastic base has been achieved by M. Shaban and M. M. [12]. Alipour. A. H. A. Hassan and Naci Kurgan [13] applied extended Kantorovich method to study buckling of skew isotropic plate on elastic foundation. Based on nonlocal elasticity theory, vibration study of non-uniform orthotropic Kirchhoff plates resting on elastic base was presented by Ma'en S. Sari and Wael G. Al-Kouz [14]. The orthotropic fluid structure coupled system has been studied by Hong Zhang et al [15] studied to analyze the structure sound and vibration behavior. Xiang Liu et al [16] offered an analytical spectral stiffness method to investigate buckling of plate resting on Winkler foundation. They combined the advantages of stiffness-based method and superposition method. A numerical technique based on finite strips and transition matrix was firstly offered by A. M. Farag [17] depending on Range Kutta method to solve the initial value problems of free and forced vibration of rectangular plates.
Since this time, the method has been applied successfully for extended work for plate [18]-[20]. To obtain the desirable effects of buckling and transverse vibration of plate, the in-plane forces with the gravitational acceleration are considered in plates. Recently, Farag [19] expressed a semi analytical solution for buckling and vibration of isotropic plate subjected to in-plane force. This technique was generalized by Farag El Sheikh [21] to achieve the analytical solution of higher order partial differential equations applied on plates. Farag's idea in his original technique was to achieve the transition matrix as a result of Range Kutta fourth order procedure achieving the accepted accuracy through a relatively lengthy numerical technique.
In the present paper the plate is divided into a limited number of equal wide strips. The transition matrix is represented analytically in a closed form to save the labor of crossing the suggested strips. Transition matrix associated with strip method is applied to transmit the effect of initial conditions across strips until final conditions are being satisfied. The present method is a closed form technique for the eigen values of clamped plate under in plane forces. Buckling and free vibration of a rectangular plate subjected to normal in-plane forces are studied in the present paper. The investigated plate is assumed rectangular, orthotropic with full clamped edges denoted by the symbol CCCC .The effects of un-axial and bi-axial in-plane forces on the natural frequency are obtained by means of the present technique. The convergence of the accuracy for the obtained results is examined. The final values are compared with those available in published literature showing good agreement.

2. MATHEMATICAL MODEL

Partial differential equation of motion for buckling and vibration of orthotropic plate under in-plane axial or biaxial forces N_x and N_y, in dimensionless form is:

$$\chi_1 \frac{\partial^4 W}{\partial \zeta^4} + 2\beta^2 \chi_2 \frac{\partial^4 W}{\partial \zeta^2 \partial \eta^2} + \beta^4 \frac{\partial^4 W}{\partial \eta^4} - \frac{N_x a^2}{D_y} \frac{\partial^2 W}{\partial \zeta^2} - \beta^2 \frac{N_y a^2}{D_y} \frac{\partial^2 W}{\partial \eta^2} + (\frac{Ka^4}{D_y})W + \frac{\overline{m} a^4}{D_y} \frac{\partial^2 W}{\partial t^2} = 0 \tag{1}$$

where, $\beta = \frac{a}{b}$ is the aspect ratio and a, b are the dimensions of plate in ζ, η directions respectively. Also, $\chi_1 = \frac{D_x}{D_y}$, $\chi_2 = \frac{H_{xy}}{D_y}$ where D_x, D_y and H_{xy} are the flexural and torsional rigidities of plate so that:

$$D_x = E_x \frac{h^3}{12(1-\nu_x \nu_y)},\ D_y = E_y \frac{h^3}{12(1-\nu_x \nu_y)},\ H_{xy} = D_x \nu_x + \frac{Gh^3}{12}$$

The other magnitudes K , $\overline{m}$, $\nu_x, \nu_y, E_x, E_y, G$ and h are respectively, modulus of subgrade, plate mass per unit area, Poisson's ratios, Young moduli, shear modulus and plate thickness. A reasonable solution for the displacement $W(\zeta,\eta,t)$ of a plate, with all edges clamped, may be represented by:

$$W(\zeta,\eta,t) = \sum_{m=1}^{M} \Psi_m(\eta)[\sin \mu_{m\zeta} - \sinh \mu_m \zeta + \alpha_m (\cos \mu_m \zeta - \cosh \mu_m \zeta)] \sin \omega t \tag{2}$$

where $\Psi_m(\eta)$ is unknown longitudinal function that satisfy the plate boundary conditions of the end supports of plate at $\eta = 0,1$ and:

$$\alpha_m = \left(\frac{\sin\mu_m - \sinh\mu_m}{\cos\mu_m - \cosh\mu_m}\right);\ \mu_1 = 4.73001,\ \mu_2 = 7.85398\ ,\ \mu_3 = 10.9955 \tag{3}$$

For a large number of m one can find that $\mu_m = (m+0.5)\pi$).

The plate is considered to be rectangular, orthotropic, with all edges clamped CCCC. Partial differential equation (1) can be reduced into the following ordinary differential equation:

$$\Psi_m'''' + \frac{1}{\beta^2}[2\chi_2\frac{c_m}{a_m} - \overline{N}_y]\Psi_m'' + \frac{1}{\beta^4}[K_G - \frac{c_m}{a_m}\overline{N}_x - \lambda_m^2 + \chi_1\frac{e_m}{a_m}]\Psi_m = 0\ .. \tag{4}$$

where: $\lambda_m^2 = \omega^2 a^4 \frac{\overline{m}}{D_y}$, $K_G = \frac{Ka^4}{D_y}$, $\overline{N}_x = \frac{N_x a^2}{D_y}$, $\overline{N}_y = \frac{N_y a^2}{D_y}$

and

$$a_m = \int_0^1 \Phi_m \Phi_m d\zeta\ , \qquad c_m = \int_0^1 \Phi_m \Phi_m'' d\zeta\ , \qquad e_m = \int_0^1 \Phi_m \Phi_m'''' d\zeta \tag{5}$$

$$\Phi_m = \sin\mu_{m\zeta} - \sinh\mu_m\zeta + \alpha_m(\cos\mu_m\zeta - \cosh\mu_m\zeta) \tag{6}$$

Conveniently Eq. (4) becomes:

$$\Psi_m'''' = [\alpha_{m1}\quad 0\quad \alpha_{m2}\quad 0]\{\Psi_m\} \tag{7}$$

where: $\alpha_{m1} = \frac{-\chi_1 e_m + c_m\overline{N}_x - K_G a_m + \lambda_m^2 a_m}{a_m\beta^4}$, $\alpha_{m2} = \frac{-2\chi_2 c_m + a_m\overline{N}_y}{a_m\beta^2}$

and $\{\Psi_m\} = [\Psi_m\quad \Psi_m'\quad \Psi_m''\quad \Psi_m''']^T$; T means transpose.

The orthotropic plate is divided into a little number N of equal orthotropic wide strips and $N+1$ nodal lines $0,1,2,....j,......,N-1,N$. So, the general solution of equation (7) is expressed at j^{th} nodal line as:

$$\{\Psi_m\}_j = [T_{k,l}]_j\ \{\Psi_m\}_{j-1};\qquad k = 1,2,3,4\quad ,l = 1,2,3,4 \tag{8}$$

where $\{\Psi_m\}_0$ and $\{\Psi_m\}_j$ are the initial vector and the j^{th} vector of plate respectively .

$[T_{k,l}]_j$ is the j^{th} transition matrix where:

$$T_{11} = 1 + \frac{A}{4!N^4} + \frac{AB}{6!N^6} + \frac{A^2 + AB^2}{8!N^8} + \frac{BA^2 + (A+B^2)BA}{10!N^{10}} + \frac{B^2A^2 + (A+B^2)(A^2 + B^2A)}{12!N^{12}}\ ,$$

$$T_{12} = \frac{1}{N} + \frac{A}{5!N^5} + \frac{AB}{7!N^7} + \frac{(A^2 + B^2A)}{9!N^9} + \frac{BA^2 + (A+B^2)BA}{11!N^{11}} + \frac{B^2A^2 + (A+B^2)(A^2 + BA)}{13!N^{13}}\ ,$$

$$T_{13} = \frac{1}{2!N^2} + \frac{B}{4!N^4} + \frac{A+B^2}{6!N^6} + \frac{AB + (A+B^2)B}{8!N^8} + \frac{(A+B^2)A + B^2A + (A+B^2)B^2}{10!N^{10}} + \frac{(A+B^2)BA + (A+B^2)(BA + (A+B^2)B)}{12!N^{12}}\ ,$$

$$T_{14} = \frac{1}{3!N^3} + \frac{B}{5!N^5} + \frac{A+B^2}{7!N^7} + \frac{BA + (A+B^2)B}{9!N^9} + \frac{(A+B^2)A + B^2A + (A+B^2)B^2}{11!N^{11}} + \frac{(A+B^2)BA + (A+B^2)(AB + (A+B^2)B)}{13!N^{13}}$$

$$T_{21}=\frac{A}{6!N^3}+\frac{AB}{5!N^5}+\frac{(A+B^2)A}{7!N^7}+\frac{BA^2+(A+B^2)AB}{9!N^9}+\frac{B^2A^2+(A+B^2)^2A}{11!N^{11}}+\frac{((A+B^2)BA+(A+B^2)(BA+(A+B^2)B))A}{13!N^{13}},$$

$$T_{22}=1+\frac{A}{4!N^4}+\frac{AB}{6!N^6}+\frac{A^2+AB^2}{8!N^8}+\frac{BA^2+(A+B^2)BA}{10!N^{10}}+\frac{B^2A^2+(A+B^2)^2A}{12!N^{12}},$$

$$T_{23}=\frac{1}{N}+\frac{B}{3!N^3}+\frac{A+B^2}{5!N^5}+\frac{AB+(A+B^2)B}{7!N^7}+\frac{A^2+2B^2A+(A+B^2)B^2}{9!N^9}+\frac{BA^2+(A+B^2)BA}{11!N^{11}}+\frac{(B^2A+(A+B^2)^2)B}{11!N^{11}}+\frac{B^2A^2+(A+B^2)^2A+(A+B^2)(B^2A+B^3A+(A+B^2)B^2)}{13!N^{13}},$$

$$T_{24}=\frac{1}{2!N^2}+\frac{B}{4!N^4}+\frac{A+B^2}{6!N^6}+\frac{AB+(A+B^2)B}{8!N^8}+\frac{AB^2+(A+B^2)^2}{10!N^{10}}+\frac{(A+B^2)BA+(A+B^2)(BA+(A+B^2)B)}{12!N^{12}},$$

$$T_{31}=\frac{A}{2!N^2}+\frac{AB}{4!N^4}+\frac{B^2(1+A)}{6!N^6}+\frac{A^2B+(A+B^2)AB}{8!N^8}+\frac{A^2B^2+(A+B^2)^2A}{10!N^{10}}+\frac{(A^2+B^2A)BA+(BA+(A+B^2)B)(A^2+B^2A)}{12!N^{12}},$$

$$T_{32}=\frac{A}{3!N^3}+\frac{AB}{5!N^5}+\frac{A^2+AB^2}{7!N^7}+\frac{BA^2+(A+B^2)AB}{9!N^9}+\frac{B^2A^2+(A+B^2)A}{11!N^{11}}+\frac{(A^2+B^2A)BA+(BA+(A+B^2)B)(A^2+B^2A)}{13!N^{13}},$$

$$T_{33}=1+\frac{A}{2!N^2}+\frac{A+B^2}{4!N^4}+\frac{AB+(A+B^2)B}{6!N^6}+\frac{AB^2+(A+B^2)^2}{8!N^8}+\frac{(A+B^2)AB+(A+B^2)(BA+(A+B^2)B)}{10!N^{10}},$$

$$T_{34}=\frac{1}{N}+\frac{A}{3!N^3}+\frac{A+B^2}{5!N^5}+\frac{AB+(A+B^2)B}{7!N^7}+\frac{AB^2+(A+B^2)^2}{9!N^9}+\frac{(A+B^2)AB+(A+B^2)(BA+(A+B^2)B)}{11!N^{11}}+\frac{(A+B^2)(A^2+B^2A)+(BA+(A+B^2)B)^2}{13!N^{13}},$$

$$T_{41}=\frac{A}{N}+\frac{AB}{3!N^3}+\frac{(A+B^2)A}{5!N^5}+\frac{[AB+(A+B^2)B]A}{7!N^7}+\frac{[AB^2+(A+B^2)^2]A}{9!N^9}+\frac{[(A+B^2)BA]A}{11!N^{11}}+\frac{[(A+B^2)(BA+(A+B^2)B)]A}{11!N^{11}}+\frac{[(A+B^2)^2A+(BA+(A+B^2)B)^2]A}{13!N^{13}},$$

$$T_{42}=\frac{A}{2!N^2}+\frac{AB}{4!N^4}+\frac{(A+B^2)A}{6!N^6}+\frac{A^2B+(A+B^2)AB}{8!N^8}+\frac{(A+B^2)A^2+(BA+(A+B^2)B)BA}{10!N^{10}}+\frac{(A+B^2)A^2B+(BA+(A+B^2)B)(A+B^2)A}{12!N^{12}},$$

$$T_{44}=1+\frac{B}{2!N^2}+\frac{A+B^2}{4!N^4}+\frac{AB+(A+B^2)B}{6!N^6}+\frac{AB^2+(A+B^2)^2}{8!N^8}$$
$$+\frac{(A+B^2)AB+(A+B^2)(BA+(A+B^2)B)}{10!N^{10}}+\frac{(A+B^2)^2A+(BA+(A+B^2)B)^2}{12!N^{12}}$$
$$T_{43}=\frac{B}{N}+\frac{A+B^2}{3!N^3}+\frac{AB+(A+B^2)B}{5!N^5}+\frac{(A+B^2)A+[A+(A+B^2)]B^2}{7!N^7}$$
$$+\frac{BA^2+(A+B^2)BA+[B^2A+(A+B^2)^2]B}{9!N^9}$$
$$+\frac{(A+B^2)A^2+[BA+(A+B^2)B]AB+[(A+B^2)BA+(A+B^2)(BA+(A+B^2)B)]B}{11!N^{11}}$$
$$+\frac{(A+B^2)A^2B+(BA+(A+B^2)B)(A+B^2)A+((A+B^2)^2A+(BA+(A+B^2)B)^2)B}{13!N^{13}},$$

and
$A=\alpha_{m1}$, $B=\alpha_{m2}$

Applying equation (8) for each orthotropic strip until the final end F of plate is reached; one can obtain the final end vector:

$$\{\Psi_m\}_F=[T_{k,l}]^N\{\Psi_m\}_0 \qquad (9)$$

If the boundary conditions at $\eta=1$ are satisfied, two characteristic equations for plate vibration will be available. The produced eigen values are the buckling natural frequencies parameters of plate.

3. INITIAL CONDITIONS
For clamped edges at $\eta=0,1$, the boundary conditions are:

$$\Psi_m=\Psi_m'=0 \quad \text{at } \eta=0,1. \qquad (10)$$

Then the initial vector $\{\Psi_m\}_0$ is expressed at $\eta=0$ as:

$$\{\Psi_m\}_0=[0 \quad 0 \quad \Delta_1 \quad \Delta_2]^T \qquad (11)$$

where Δ_1 and Δ_2 are two arbitrary constants. Applying the boundary conditions for the final clamped edge of plate at $\eta=1$, in Eq. (10), one can get:

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}\begin{Bmatrix}\Delta_1\\ \Delta_2\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \qquad (12)$$

The coefficients s_{rq}; $r=1,2$, $q=1,2$ match the properties of plates and the coefficient of restraint against rotation. The eigen values of the obtained characteristic equations in Eq. (12) are the buckling natural frequency parameters of plate.

4. RSULTS AND DISCUSIONS
The present technique is applied to a rectangle orthotropic clamped plate with various parameters of aspect ratio, orthotropic properties and in-plane forces. The closed form solutions are achieved analytically and expressed graphically for some cases. To verify the validity of obtained results comparisons are made with the available results.

The fundemantal buckling natural frequency parameters $\lambda = \omega a^2 \sqrt{\frac{\overline{m}}{D_y}}$ is presented in Table: 1 for orthotropic rectangular clamped plates CCCC with $\beta = 1.5$ under selected range of a parameter $\Re_x$ of the noralized in-plane forces $\overline{N}_x = \Re_x \beta^2 / 6$, $\overline{N}_y = 0$. The results are obtained for three cases of orthotropic plates and compared with available in Farag [19]. Compressions agree well showing a good accuracy of the present technique.

Table: 1 Fundemantal buckling natural frequency parameters λ for orthotropic rectangular clamped plates CCCC with $\beta = 1.5$ under noralized in-plan forces $\overline{N}_x = \Re_x \beta^2 / 6$, $\overline{N}_y = 0$

$\Re_x$	$\frac{D_x}{2} = \frac{D_y}{2} = H_{xy}$	$D_x = \frac{D_y}{2} = H_{xy}$	$D_x = D_y = H_{xy} = 1$	
	Present		**Present**	**[19]**
-100	53.90940483	51.57186408	56.93095169	56.930958
-50	55.99307719	53.74623649	58.90784347	58.907850
0	58.00194360	55.83599826	60.82051297	60.820519
50	59.94352529	57.85031951	62.67483993	62.674845
100	61.82416187	59.79682459	64.47585846	64.475864

Buckling natural frequency parameters $\lambda = \omega a^2 \sqrt{\frac{\overline{m}}{D_y}}$ for orthotropic square clamped plates CCCC under normalized compressive forces are presented in Fig. 2. The results are achieved for the first two modes under different values of rigidities parameters $\chi_1 = \frac{D_x}{D_y}$, $\chi_2 = \frac{H_{xy}}{D_y}$ and bulking forces $\overline{N}_x$, $\overline{N}_y$.

Table: 2 Buckling natural frequency parameters λ for orthotropic square clamped plates CCCC under normalized compressive forces $\overline{N}_x$, $\overline{N}_y$ according to various magnitudes of rigidity parameters

Rigidity Parameters		**Buckling Forces**		**Natural FrequencyParameters**	
χ_1	χ_2	$\overline{N}_x$	$\overline{N}_y$	**First mode**	**Second mode**
1.00	0.25	-100	-25	46.85729028	102.3609451
1.00	0.50	-100	-50	36.20709705	92.84404767
1.00	0.75	-100	-75	20.35962007	82.26573605
1.00	1.00	-100	-100	70.17437804	623.6069342
1.00	1.00	-50	-100	10.63488466	74.39833348
1.00	1.25	-50	-50	11.84741162	52.72171773
1.00	1.50	-25	-25	28.82019705	67.24924924
1.00	1.75	0	-25	34.78294395	71.46765250
1.00	2.00	0	-50	31.29830247	65.11093725

The closed form solutions are carried out for many cases of clamped plates. The implicit formulae of the bulking and vibration eigen values λ are achieved and represented graphically for each case:

Case 1: The present technique is applied to a square orthotropic clamped plate with $D_x = 0.25 D_y$, $H_{xy} = D_y$ under external in-plane forces $\overline{N}_x = -30$, $\overline{N}_y = 0$, the implicit closed form solution of λ is expressed by:

$$f(\lambda) = -0.000477391\lambda^2 + .22446307 + 2.25299(10)^{-30}\lambda^{14} + 5.80232(10)^{-35}\lambda^{16} + 1.0500063(10)^{-39}\lambda^{18} + 1.23212959(10)^{-44}\lambda^{20} + 5.69064463(10)^{-50}\lambda^{22} + 1.476790739(10)^{-55}\lambda^{24} + 1.3632874(10)^{-7}\lambda^{4} - 1.1034028(10)^{-11}\lambda^{6} + 3.467277(10)^{-16}\lambda^{8} - 4.58029(10)^{-21}\lambda^{10} - 6.09863(10)^{-26}\lambda^{12} \quad (13)$$

The natural frequency parameters λ is evaluated where $f(\lambda)=0$ (See Fig. 1) . It is obtained in the first third modes such as: $\lambda = 23.53631472, \; 68.38524146$ and 127.8990314

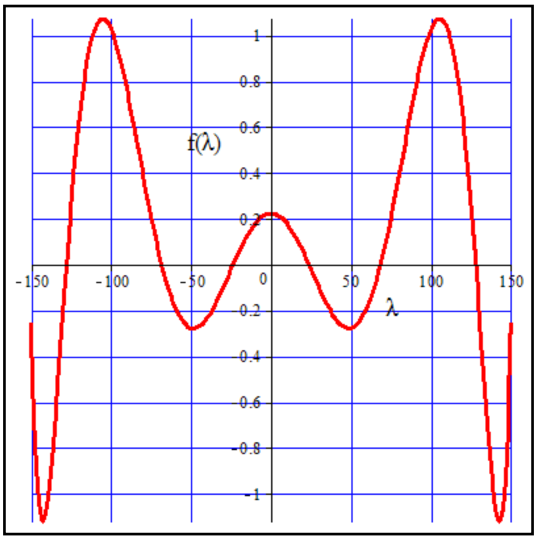

Fig. 1 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where $D_x = 0.25D_y$, $H_{xy} = D_y$, $\overline{N}_x = -30$, $\overline{N}_y = 0$, $K_G = 0$

Case 2: A rectangular orthotropic clamped plate with $a = 1.5b$, $D_x = 2D_y$, $H_{xy} = 2D_y$ under external in-plane forces $\overline{N}_x = -30$, $\overline{N}_y = 0$, is studied and the natural frequency parameters λ is expressed by:

$$f(\lambda) = -0.000103468632\lambda^2 + 0.364163764 + 2.301758(10)^{-35}\lambda^{14} + 1.056491(10)^{-40}\lambda^{16} + 3.701264(10)^{-46}\lambda^{18} + 9.4649632(10)^{-52}\lambda^{20} + 9.41994310(10)^{-58}\lambda^{22} + 5.211209402(10)^{-64}\lambda^{24} + 5.3444022(10)^{-9}\lambda^{4} - 8.32331(10)^{-14}\lambda^{6} + 5.12073(10)^{-19}\lambda^{8} - 1.29932(10)^{-24}\lambda^{10} - 3.8470(10)^{-30}\lambda^{12} \quad (14)$$

The rail and imaginary values of λ is evaluated where $f(\lambda)=0$ (See Fig. 2). It is obtained in the first third modes such as: $\lambda = 67.00806235, \; 157.7928519$ and 258.3219110

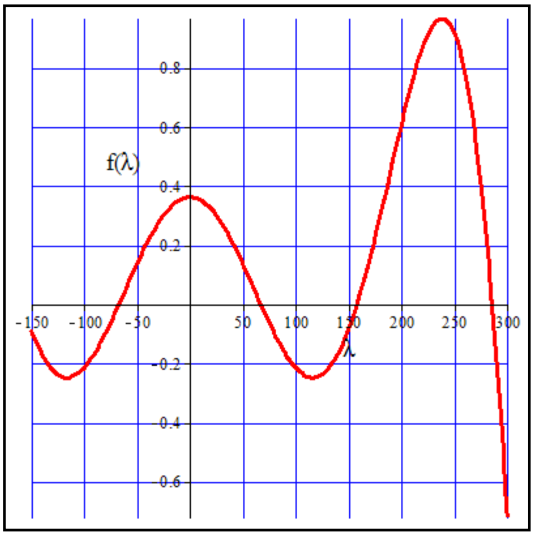

Fig. 2 Natural frequency parameter λ for orthotropic rectangular clamped plate under in-plane force where $D_x = 2D_y$, $H_{xy} = 2D_y$, $\bar{N}_x = -30$, $\bar{N}_y = 0$, $a = 1.5b$, $K_G = 0$

Case 3: An orthotropic clamped rectangular plate with $a = 1.25b$, $D_x = 2D_y$, $H_{xy} = 0.5D_y$ under external in-plane forces $\bar{N}_x = \bar{N}_y = -40$ and resting on homogenous subgrade with $K_G = 1$ is studied and the natural frequency parameters λ is expressed by:

$$
\begin{aligned}
f(\lambda) &= -0.4122777779(10)^{-4}\lambda^2 + 0.04003995681 + 2.496750(10)^{-33}\lambda^{14} \\
&-1.242684(10)^{-38}\lambda^{16} - 2.4582382(10)^{-43}\lambda^{18} - 6.5478937(10)^{-49}\lambda^{20} \\
&+ 6.3086394(10)^{-55}\lambda^{22} + 3.293353397(10)^{-60}\lambda^{24} + 6.49968513(10)^{-9}\lambda^{4} \\
&- 2.691897(10)^{-13}\lambda^{6} + 4.20418(10)^{-18}\lambda^{8} - 3.52729(10)^{-23}\lambda^{10} - 1.41364(10)^{-28}\lambda^{12}
\end{aligned}
\qquad (15)
$$

The rail and imaginary values of λ is evaluated where $f(\lambda) = 0$ (See Fig. 3) . It is obtained in the first third modes such as: $\lambda = 34.35497010,\ 88.10823812$ and 167.7041678

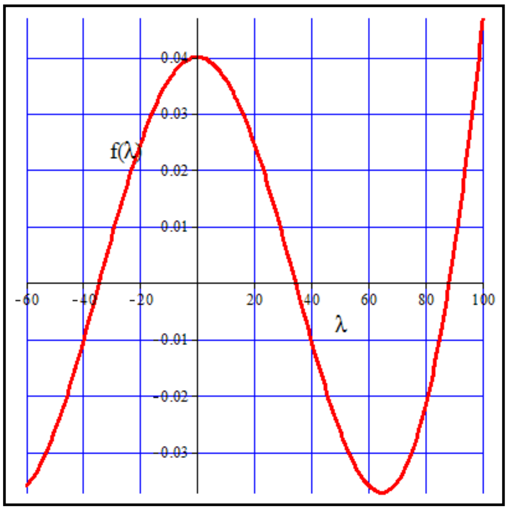

Fig. 3 Natural frequency parameter λ for orthotropic rectangular clamped plate under in-plane force where $D_x = 2D_y$, $H_{xy} = 0.5D_y$, $\bar{N}_x = \bar{N}_y = -40$, $a = 1.25b$, $K_G = 1$

Case 4: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane forces $\bar{N}_x = \bar{N}_y = -40$ and resting on homogenous subgrade with $K_G = 500$ is studied and the closed form solution of λ is expressed by:

$$f(\lambda) = -0.6352271609\,(10)^{-4}\lambda^2 + 0.0216122228 + 1.125277(10)^{-30}\lambda^{14}$$
$$-2.389763(10)^{-35}\lambda^{16} - 9.456019(10)^{-40}\lambda^{18} - 7.0538636(10)^{-45}\lambda^{20}$$
$$+4.1793584(10)^{-51}\lambda^{22} + 1.476790739(10)^{-55}\lambda^{24} + 2.75551644(10)^{-8}\lambda^{4} \qquad (16)$$
$$-2.991358(10)^{-12}\lambda^{6} + 1.199433(10)^{-16}\lambda^{8} - 2.81865(10)^{-21}\lambda^{10} - 2.20548(10)^{-26}\lambda^{12}$$

Value of λ is evaluated where $f(\lambda)=0$ (See Fig. 4) . It is obtained in the first third modes such as: $\lambda = 20.24678014,\ \ 52.59953663$ and 102.8995049

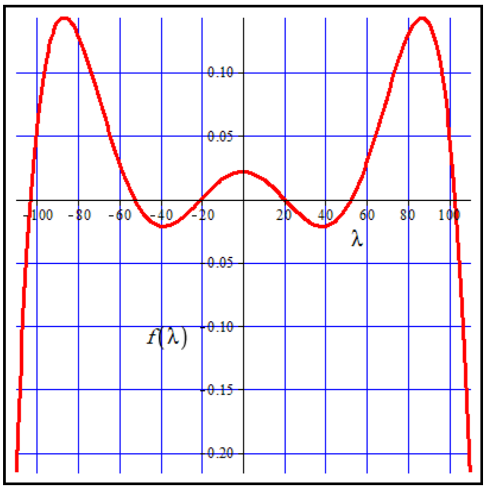

Fig. 4 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $\overline{N}_x = \overline{N}_y = -40$, , $K_G = 500$

Case 5: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane forces $\overline{N}_x = \overline{N}_y = -40$ and resting on homogenous subgrade with $K_G = 600$ is studied and the natural frequency parameters λ is expressed by:

$$f(\lambda) = -0.00006912397079\lambda^2 + 0.02824304892 + 1.144056(10)^{-30}\lambda^{14}$$
$$-2.304975(10)^{-35}\lambda^{16} - 9.385457(10)^{-40}\lambda^{18} - 7.0583634(10)^{-45}\lambda^{20}$$
$$+4.0021435(10)^{-51}\lambda^{22} + 1.476790739(10)^{-55}\lambda^{24} + 2.845979664(10)^{-8}\lambda^{4} \qquad (17)$$
$$-3.039617(10)^{-12}\lambda^{6} + 1.213493(10)^{-16}\lambda^{8} - 2.80518(10)^{-21}\lambda^{10} - 2.28491(10)^{-26}\lambda^{12}$$

The natural frequency parameters λ is evaluated where $f(\lambda)=0$ (See Fig. 5) . It is obtained in the first third modes such as: $\lambda = 22.58167634,\ 53.54167766$ and 103.3842740

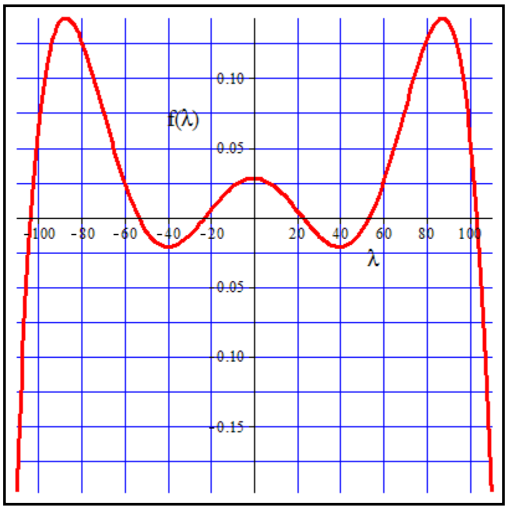

Fig. 5 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $\overline{N}_x = \overline{N}_y = -40$, , $K_G = 600$

Case 6: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane forces $\overline{N}_x = \overline{N}_y = -40$ and resting on homogenous subgrade with $K_G = 800$ is studied and the natural frequency parameters λ is expressed by:

$$\begin{aligned} f(\lambda) &= 1.476790739\,(10)^{-55}\lambda^{24} - 2.137309\,(10)^{-35}\lambda^{16} - 9.244205\,(10)^{-40}\lambda^{18} \\ &- 7.0667783\,(10)^{-45}\lambda^{20} + 3.6477138\,(10)^{-51}\lambda^{22} + 1.179591(10)^{-30}\lambda^{14} \\ &- 2.447609\,(10)^{-26}\lambda^{12} + 3.031291457\,(10)^{-8}\lambda^{4} - 3.13781518\,(10)^{-12}\lambda^{6} \\ &+ 1.2414052\,(10)^{-16}\lambda^{8} - 2.776795\,(10)^{-21}\lambda^{10} - 0.8087654902\,(10)^{-4}\lambda^{2} \\ &+ 0.04323074694 \end{aligned} \tag{18}$$

The natural frequency parameters λ is evaluated where $f(\lambda) = 0$ (See Fig. 6). It is obtained in the first third modes such as: $\lambda = 26.64455117$, 55.37789497 and 104.3470561

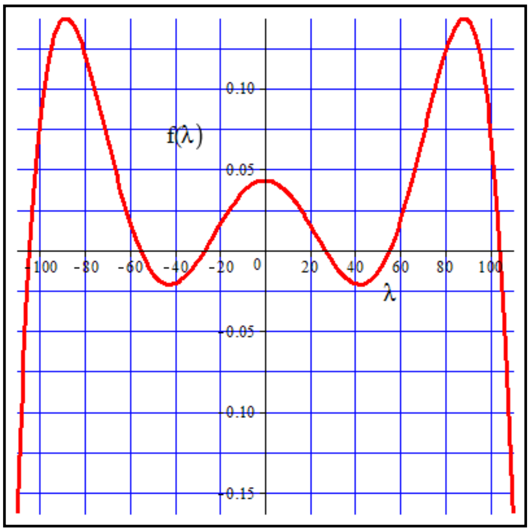

Fig. 6 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $\overline{N}_x = \overline{N}_y = -40$, , $K_G = 800$

Case 7: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane forces $\overline{N}_x = \overline{N}_y = -40$ and resting on homogenous subgrade with $K_G = 900$ is studied and the closed form solution of λ is expressed by:

$$\begin{aligned} f(\lambda) = &-0.8703376446(10)^{-4}\lambda^2 + 0.05162468132 + 1.196358(10)^{-30}\lambda^{14} \\ &- 2.054426(10)^{-35}\lambda^{16} - 9.173517(10)^{-40}\lambda^{18} - 7.0706931(10)^{-45}\lambda^{20} \\ &+ 3.4704989(10)^{-51}\lambda^{22} + 1.476790739(10)^{-55}\lambda^{24} + 3.126173541(10)^{-8}\lambda^4 \\ &- 3.18774858(10)^{-12}\lambda^6 + 1.2552520(10)^{-16}\lambda^8 - 2.761860(10)^{-21}\lambda^{10} \\ &- 2.530766(10)^{-26}\lambda^{12} \end{aligned} \quad (19)$$

The natural frequency parameters λ is evaluated where $f(\lambda) = 0$ (See Fig. 7) . It is obtained in the first third modes such as: $\lambda = 28.45930616,\ 56.27353957$ and 104.8251311

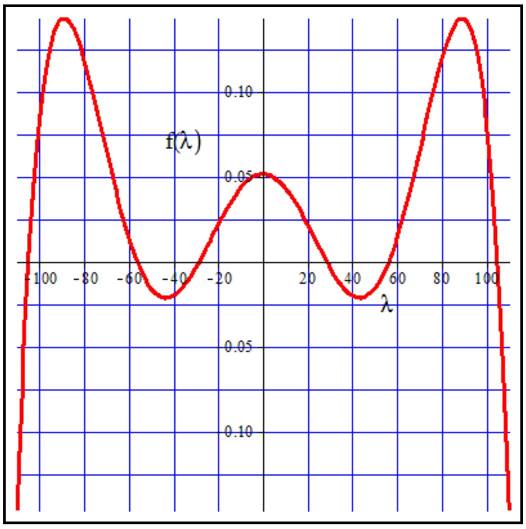

Fig. 7 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$, $\overline{N}_x = \overline{N}_y = -40$, , $K_G = 900$

Case 8: A square orthotropic clamped plate with $D_x = 0.5D_y$, $H_{xy} = 0.5D_y$ under external in-plane force $N_x = N_y = -40$ and resting on homogenous subgrade with $K_G = 1000$ is studied and the closed form solution of λ is expressed by:

$$\begin{aligned} f(\lambda) = &-0.9338224739(10)^{-4}\lambda^2 + 0.0606438753 + 1.212464(10)^{-30}\lambda^{14} \\ &-1.972185(10)^{-35}\lambda^{16} - 9.102791(10)^{-40}\lambda^{18} - 7.0744133(10)^{-45}\lambda^{20} \\ &+ 3.2932839(10)^{-51}\lambda^{22} + 1.476790739(10)^{-55}\lambda^{24} + 3.222561906(10)^{-8}\lambda^4 \\ &- 3.238234(10)^{-12}\lambda^6 + 1.26902(10)^{-16}\lambda^8 - 2.7464(10)^{-21}\lambda^{10} - 2.61507(10)^{-26}\lambda^{12} \end{aligned} \quad (20)$$

The natural frequency parameters λ is evaluated where $f(\lambda) = 0$ (See Fig. 8). It is obtained in the first third modes such as: $\lambda = 30.16508090,\ 57.15515081$ and 105.3010357

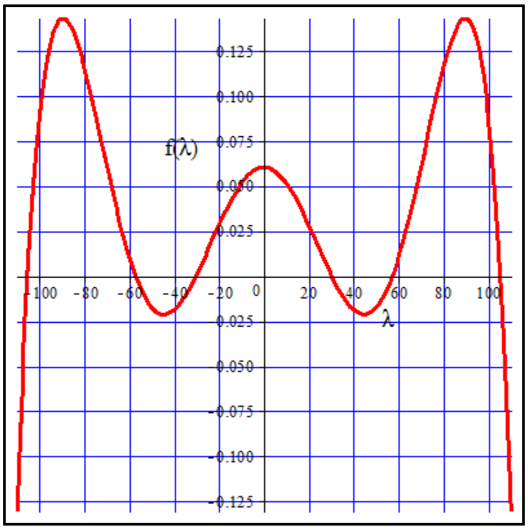

Fig. 8 Natural frequency parameter λ for orthotropic square clamped plate under in-plane force where

$$D_x = 0.5D_y \;\; , H_{xy} = 0.5D_y , \overline{N}_x = \overline{N}_y = -40 \;\; , K_G = 1000$$

5. CONCLUSION

A combination between the series expansion and strip method is presented here to derive a closed form solutions for eigen values on buckling and vibration of plates subjected to in-plane forces. The plate under study is clamped orthotropic rectangular resting on elastic homogenous sub-grade base. On the present method the plate domain is divided into a limited number of wide strips (panels) to be solved by the power series expansion. A limited number of strips can be applied with increasing the number of terms of the expanded series to preserve a high accuracy in the achieved solution. Many cases of orthotropic plates under different values of elastic foundation parameters and aspect ratios are investigated by the present technique. The achieved method is illustrated and the accuracy is verified via several numerical examples examining buckling and vibration of orthotropic plate under the unaxial and biaxial in-plane forces and elastic coefficients of subgrade. The study shows a good agreement in comparisons which prove the validity and applicability of the present technique.

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