

SEMIGROUPS AND FORMAL LANGUAGES

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Abstract

The underlying principle is always the same: Given an “alphabet” or “vocabulary” (consisting of letters, words, punctuation symbols, number symbols, programming instructions, etc.), we have a method (grammar) for constructing “meaningful” words or sentences from this alphabet. This immediately reminds us of the term “word semigroup” and indeed, these free semigroups will play a major role, the language L constructed will be a subset of the free semigroup A^* or the monoid A^* on the alphabet A .

Key words: Semigroup, Grammar, Automata.

Introduction:

Besides the natural languages such as English, so-called formal languages are of importance in the formal sciences, especially in computing or information science.

There are essentially three ways to construct a language L .

- (i) Approach via grammar: given a collection of rules (grammar), generate L from A .
- (ii) Approach via automata : consider an initial semi automation which processes the elements of L in a suitable way.
- (iii) Algebraic approach : L is constructed by the algebraic combination of certain subsets of A .

All three approaches use algebraic method. Let us start with method (i) This approach is largely based on the work of Chomsky (1957).

1.1 DEFINITION:

A phase-structure grammar is a quadruple $G = (A, G, \rightarrow, g_0)$, where A and G are nonempty disjoint finite sets, $g_0 \in G$ and \rightarrow is a finite relation from G into $(A \cup G)^*$.

1.2 DEFINITION:

Let G be a grammar.

- (i) A is called the alphabet of G .
- (ii) G is called the set of grammar symbols.
- (iii) $V := A \cup G$ is the complete vocabulary.
- (iv) The elements (x, y) in \rightarrow (which is a subset of $G \times V^*$) are also written in the form $x \rightarrow y$ and are called productions or rewriting rules.
- (v) g_0 is called the initial symbol.

We now obtain a suitable subset of A^* , by using G . In order to do this, we need another relation, this time on V^* .

1.3 DEFINITION:

Let $G = (A, G, \rightarrow, g_0)$ be a grammar. For $y, z \in V^*$, we write $y \Rightarrow z$ if there are $v \in G$ and $v_1, v_2, w \in V^*$ with

$$Y = v_1 v v_2, z = v_1 w v_2, \text{ and } v \rightarrow w.$$

The reason for introducing \Rightarrow is to obtain a new word $v_1, v_2 \dots v'_r \dots v_n$ from a given word $v_1 v_2 \dots v_r \dots v_n$ and a rule $v_r \rightarrow v'_r$. Thus we extend \rightarrow to a compatible relation \Rightarrow on

V^* . Then transitive hull is $y \Rightarrow t \text{ of } \Rightarrow$ is given by $y \Rightarrow t z \Leftrightarrow$ there are $n \in \mathbb{N}_0$ x_0, \dots, X_n
 $\in V^*$

With $y = x_0 \Rightarrow \dots \Rightarrow X_n = z$.

The sequence x_0, \dots, X_n is called a derivation of z from y ; n the length of the derivation.

1.4 DEFINITION:

Let $G = (A, G, \rightarrow, g_0)$ be a grammar

$$L(G) := \{ l \in A^* \mid g_0 \Rightarrow t l \}$$

is called the language generated by G (also the phase-structure language).

1.5 DEFINITION:

A grammar $G = (A, G, \rightarrow, g_0)$ and its generated language are called right linear if all elements in \rightarrow are of the forms $g \rightarrow a$ or $g \rightarrow ag'$. With $g, g' \in G$, $a \in A^*$. G is called context-free if for all $x \rightarrow y$ in \rightarrow we have $l(x) = l$.

In a right linear language, the length of a word is never shortened by any derivation. All right linear grammars are context-free.

1.6 DEFINITION:

Let A be a set and $R \subseteq A^*$. R is called regular or a regular language if R can be obtained from one-element subset of A^* by a finite number of admissible operations. Admissible operations are the formations of unions, products, and generated submonoids.

The beginning of the study of formal languages can be traced to Chomsky, who in 1957 introduced the concept of a context-free language in order to model natural languages. Since the late 1960s there has been considerable activity in the theoretical development of context-free

languages both in connection with natural languages and with the programming languages. Chomsky used Semi-Thue systems to define languages, which can be described as certain subjects of finitely generated free monoids. Chomsky (1957) details a revised approach in the light of experimental evidence and careful consideration of semantic and syntactic structure of sentences.

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