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A NEW CHARACTERIZATION OF SEMI OPEN SET IN TOPOLOGICAL SPACE

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ABSTRACT

This paper presents the study of some new results on properties of open set in Topological space. Here, we consider usual Topology R which is neither open nor closed in R and here, it is proved that the set is semi open but not open, closed, pre-open or pre-closed.

KEYWORDS :

Discrete Space , Open Set, Semi – open set, Union set. Interior, Closure, Complement, Topological space

INTRODUCTION :

Bahan, R (1) and Prasad, D (6) are the pioneer workers of the present area. In fact, the present work is the extension of work done by Ferrer, J(2), Jha M.N. et al (3), Kelly et al (4), Levine. N(5) and Santi Leela et al (7). In this paper, we have studied a new characterization of semi open set in Topological space.

Here, we use the following Notations, definitions and Fundamental Ideas.

Notations

Int.(cl(A)) ≡ The Interior of the closure of the set

$\frac{0}{A}$ ≡ The Interior of the closure of the set A.

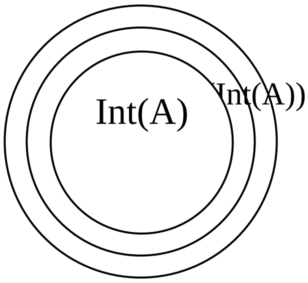
Cl(Int.(A)) ≡ The closure of the interior of A.

A^c ≡ Complement of the set A.

DEFINITION

1. **SEMI-OPEN SET :**

Let $A \subseteq X$, a topological space. The A is a semi-open set if $A \subseteq Cl(Int.(A))$.



Example (1.1) :

Let A= (a ,b) in R with usual topology, which is neither open nor closed in R.

Proof:

Since Int (A) = (a, b)

$\Rightarrow Cl(Int.(A)) = Cl((a, b)) = [a, b] \supseteq A$

$\Rightarrow A \subseteq Cl(Int.(A))$

Therefore A is semi-open.

So a semi-open set may not be open or closed.

Further

Since $Cl(Int(A)) \not\subseteq A$

Therefore A is not pre-closed.

Also Cl(A) = [a, b]

$\Rightarrow Int(cl(A)) = (a, b) \not\supseteq A$

$\Rightarrow A \not\subseteq Int(cl(A))$

Therefore A is not pre-open.

Thus a set may be semi open but not open, closed, pre-open or pre-closed.

Example (1.2) :

Let A = (a, b] be in R with usual topology then A is semi-open.

Proof:

Since Int(A) = (a, b)

$\Rightarrow Cl(Int(A)) = [a, b] \supseteq A$

$\Rightarrow A \subseteq Cl(Int(A))$

Therefore A is semi-open

Example (1.3) :

Consider discrete space X. Since every subset is open, it is semi-open.

Example (1.4) :

Let X = {1, 2, 3}

T = {ϕ, X, {1}, {2}}

Let A = {1, 3}

Then Int(A) = {1}

$\Rightarrow$ Cl(Int(A)) = {1, 2, 3} = X $\supseteq$ A

$\Rightarrow$ A $\subseteq$Cl(Int(A))

Therefore A is semi-open.

It is also a pre-open set.

Example (1.5) :

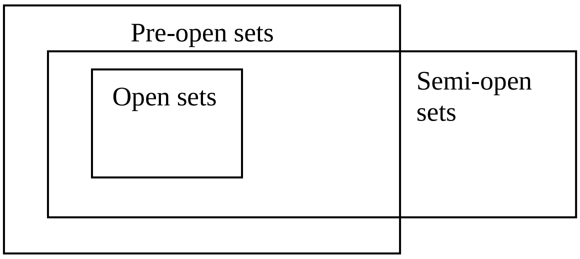
Consider indiscrete space X and $\phi \subset A \subset X$.

Since Int(A) = ϕ

$$\Rightarrow Cl(Int(A)) = \phi \not\supseteq A$$

$$\Rightarrow A \not\subseteq Cl(Int(A))$$

Therefore A is not semi-open.



2. MATHEMATICAL TREATMENT OF THE PROBLEM:-

2.1 Let A be semi-open. Then there exists an open set G such that

$$G \subseteq A \subseteq \overline{G}$$

Proof:

Let A be semi-open, then $A \subseteq \overline{IntA}$

Therefore $IntA \subseteq A \subseteq \overline{IntA}$

Let G = IntA, then G is open.

Also $G \subseteq A \subseteq \bar{G}$ (By Using Property)

2.2 An open set is semi-open.

Proof:

Let A be open.

Then A = Int(A)

$\Rightarrow A \subseteq cl(Int(A))$

Therefore A is semi-open

2.3 A' is a semi open set $\Leftrightarrow$ A is semi-closed set.

Proof:

Let A' be a semi-open set, then

$A' \subseteq \overline{IntA'}$

But $\bar{A} = (IntA')'$

Hence $\overline{IntA'} = \left(Int(IntA')'\right)'$

Therefore $A' \subseteq \left(Int(IntA')'\right)'$

$\Rightarrow A \supseteq Int(IntA')' = Int\bar{A}$

Therefore A is a semi-closed set.

Since all steps can be reversed.

So A is semi-closed $\Rightarrow A'$ is semi-open.

Corollary (2.1)

A is semi-open set $\Leftrightarrow A'$ is a semi-closed set.

2.4 Union of an arbitrary family of semi-open sets is semi-open.

Proof:

Let $\{A_i\}$, $i \in I$ be a family of semi-open sets.

Then $A_i \subseteq \overline{IntA_i}, \forall i$

Now $A_i \subseteq \bigcup_i A_i$

$\Rightarrow Int(A_i) \subseteq Int(\cup A_i)$

$\Rightarrow \overline{IntA_i} \subseteq \overline{Int(\cup A_i)}$

Therefore $A_i \subseteq \overline{Int(\cup A_i)}, \forall i$

$\Rightarrow \cup A_i \subseteq \overline{Int(\cup A_i)}$

Therefore $\cup A_i$ is semi-open.

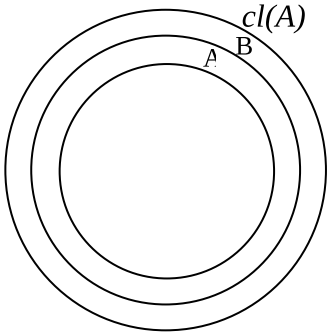
Main Result

Let A be semi-open in the topological space X and suppose $A \subseteq B \subseteq cl(A)$. *Then B is semi-open.*

Proof :

There exists an open set G such that $G \subseteq A \subseteq cl(G)$.

Now $A \subseteq B$



Therefore $G \subseteq A \subseteq B$

$\Rightarrow G \subseteq B$

But $A \subseteq cl(G)$

$\Rightarrow cl(A) \subseteq cl(G)$

and thus $B \subseteq cl(A) \subseteq cl(G)$

Therefore $G \subseteq B \subseteq cl(G)$

Hence B is semi-open.

Hence the Result.

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