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Study of History of Ancient Indian Mathematics and its Impact

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Abstract

In the classical period of Indian mathematics (400 CE to 1600 CE), important contributions were made by scholars like Aryabhata, Brahmgupta, Mahavira, Bhaskara II, Madhava of Sangamagrama and Nilakantha Somayaji. The decimal number system in worldwide use today was first recorded in Indian mathematics. Indian mathematicians made early contributions to the study of the concept of zero as a number, negative numbers, arithmetic, and algebra. In addition, trigonometry was further advanced in India, and, in particular, the modern definitions of sine and cosine were developed there. These mathematical concepts were transmitted to the Middle East, China, and Europe and led to further developments that now form the foundations of many areas of mathematics. There was an awareness of ancient Indian mathematics in the West since the sixteenth century; historians discuss the Indian mathematical tradition only after the publication of the first translations by Colebrooke in 1817. Its reception cannot be comprehended without accounting for the way new European mathematics was shaped by Renaissance humanist writings. We show by means of a case study on the algebraic solutions to a linear problem how the understanding and appreciation of Indian mathematics was deeply influenced by humanist prejudice that all higher intellectual culture, in particular all science, had risen from Greek soil.

Keywords: Indian mathematics, renaissance, humanism, Greek, Indian influences

Introduction

Indian mathematics emerged in the Indian subcontinent from 1200 BCE until the end of the 18th century. In the classical period of Indian mathematics (400 CE to 1600 CE), important contributions were made by scholars like Aryabhata, Brahmgupta, Mahavira, Bhaskara II, Madhava of Sangamagrama and Nilakantha Somayaji. The decimal number system in worldwide use today was first recorded in Indian mathematics. Indian mathematicians made early contributions to the study of the concept of zero as a number, negative numbers, arithmetic, and algebra. In addition, trigonometry was further advanced in India, and, in particular, the modern definitions of sine and cosine were developed there. These mathematical concepts were transmitted to the Middle East, China, and Europe and led to further developments that now form the foundations of many areas of mathematics. While there was an awareness of ancient Indian mathematics in the West since the sixteenth century, historians discuss the Indian mathematical tradition only after the publication of the first translations by Colebrooke in 1817. Its reception cannot be comprehended without accounting for the way new European mathematics was shaped by Renaissance humanist writings. Western reception of ancient Indian mathematics during the nineteenth century is very much biased by the humanist tradition. Reflections and statements of Western historians on Indian mathematics can only be fully understood if this context is known and acknowledged. During the Middle Ages mathematics was hardly practiced or appreciated by the intellectual elite. The middle ages knew two traditions of mathematical practice. On the one hand, there was the scholarly tradition of arithmetic theory, taught at universities as part of the quadrivium. The basic text on arithmetic, presented as one of the seven liberal arts, was Boethius's De Institutione Arithmetica (Friedlein, 1867). The Boethian arithmetic strongly relies on Nichomachus of Gerasa's Arithmetica from the

2nd century. This basically qualitative arithmetic deals with properties of numbers and ratios. All ratios have a name and operations or propositions on ratios are expressed in a purely rhetorical form. The qualitative aspect is well illustrated by the following proposition from Jordanus de Nemore's De elementis arithmetice artis (c. 1250, Book IX, proposition LXXI; Busard 1991, 199). During the eleventh century a board game named Rhythmomachia was designed to meet with these aesthetic aspirations. Originated as the subject of a competition on the knowledge of Boethian arithmetic amongst cathedral schools in Germany, the game was played until the sixteenth century, when the arithmetic tradition passed into oblivion. Despite its limited applicability, Boethian arithmetic evolved into a specific kind of mathematics, typical for the European Middle Ages, and left its mark on early natural philosophy. Arithmetical problem solving became much more advanced with the introduction of Arabic algebra through the Latin translations of al Khwarizmi's Algebra by Robert of Chester (c. 1145), Gerard of Cremona (c. 1150) and Guglielmo de Lunis (c. 1215). During the fifteenth century Italian humanists eagerly started collecting editions of Greek mathematics; one of the most industrious was Cardinal Bessarion. In some sense Wallis's Treatise on Algebra (1685) can be considered the first serious historical investigation of the history of algebra. John Wallis was well informed about Arabic writings through Vossius and was one of the first to attribute correctly the name algebra to al - jabr in Kitab fi al - jabr wa'l - muqabala.

He also pointed out the mistaken origin of algebra as Geber's name, which was a common misconception before the seventeenth century. So, while in the seventeenth century no Sanskrit mathematics had yet been introduced into Europe, scholars by then were aware of the existence of Indian algebra. Wallis's view persisted in eighteenth-century historical studies, which reiterated the influence from Indian mathematics. In the early nineteenth century, the English orientalist Henry Thomas Colebrooke, who previously published his Sanskrit Grammar (1805), undertook the task of translating three classics of Indian mathematics, the Brahmasphut asiddhanta of Brahmagupta (628) and the Lilavati and the Bijaganita of Bhaskara II (1150). At once European historians had something to reflect upon. In a period when mathematics was hardly practiced in Europe and in the Islam regions, there appeared to have existed this Indian tradition in which algebraic problems were solved with multiple unknowns, in which zero and negative quantities were accepted and in which sophisticated methods were used to solve indeterminate methods.

In general, nineteenth-century historians showed an admiration for the Hindu tradition. However, whenever explanations were required, scholars became divided into two opposing camps, Non believers did not grant Indian mathematicians the status of original thought. Indian knowledge must have stemmed from the Greeks, the cradle of Western mathematics, or even mathematics as such. The major non-believer was Moritz Cantor who published an influential four-volume work on the history of mathematics (1880-1908). Cantor (1894, II) takes every opportunity to point out the Greek influences on Hindu algebra. Soon after Kern (1875) published the Sanskrit edition of the Aryabhatiya (AB), the French orientalist L´eon Rodet was the first to provide a translation in a Western language (1877, published in Rodet 1879). Rodet wrote several articles and monographs on Indian mathematics and its relation with earlier and later developments in the Arab and Western world, published in the French Journal Asiatiques. He is the scholar who displays the most balanced and subtle views on the relations between traditions. In particular, his appraisal of Hindu and Arabic algebra as two independent traditions is still of value today. He certainly was a believer. Concerning Aryabhata's inadequate approximation of the volume of a sphere (prop. 7), he writes somewhat cynically that if Aryabhata got his knowledge from the Greeks, then apparently he chose to ignore Archimedes ("Mais elle a, pour l'histoire des math´ematiques, d'autant plus de valeur, parce qu'elle nous d´emonstre que si Aryabhat.a avait re¸cu quelque enseignement des Grecs, il ignorait au moins les travaux d'Archim`ede", Rodet 1879, 409). George Thibaut who translated several Sanskrit works on astronomy, such as Varahamihira's Pancasiddhantika (1889), also wrote an article on Indian mathematics and astronomy in the Encylopedia of Indo-Aryan Research. Concerning influences from Greek mathematics, he takes a middle position. In discussing Hindu algebra he writes that "in all these correspondences does Indian algebra surpass Diophantus" ("In allen diesen Beziehungen erhebt sich die indische algebra erheblich ¨uber das von Diophant Geleistete". As on the origins of Indian mathematics, he points out that Indian algebra, especially indeterminate analysis, is closely intertwined with its astronomy. As he argued on the Greek roots of Indian "scientific" astronomy, his evaluation is that Indian mathematics is influenced by the Greeks through astronomy. However, he adds that several arithmetical and algebraic methods are truly Indian.

The first Indian source for a formulation of this rule is from Aryabhata I, as follows: If you know the results obtained by subtracting successively from a sum of quantities each one of these quantities set these results down separately. Add them all together and divide by the number of terms less one. The result will be the sum of all the quantities. The rule is somewhat obscure and difficult to understand without examples, but some observations can be drawn from the formulation which is central to our further discussion. Firstly, the rule is valid for any number of quantities. It is not limited to two or three quantities. Secondly, the sum of all the quantities is unknown and is provided by the rule. Furthermore, and not evident from the rule, as cited above, is that the partial sums relate to the total of all the quantities, except one. For example suppose n amounts (a_1, a_2, ..., a_n) with unknown sum S and with the partial sums (s_1, s_2, ..., s_n) given, where si = S − ai, then

$$S = \frac{\sum_{i=1}^{n} s_i}{n-1}$$

The rule and the problems it applies to should not be confused with a similar problem in which the partial sums of two consecutive quantities are given. For three numbers, the problems are evidently the same, but they diverge for more than three quantities. E.g., for five quantities the corresponding equations are:

$a_1 + a_2 + a_3 + a_4 = s_1$		$a_1 + a_2 = s_1$
$a_1 + a_3 + a_4 + a_5 = s_2$		$a_2 + a_3 = s_2$
$a_1 + a_2 + a_4 + a_5 = s_3$	and	$a_3 + a_4 = s3$
$a_1 + a_2 + a_3 + a_5 = s_4$		$a_4 + a_5 = s4$
$a_2 + a_3 + a_4 + a_5 = s_5$		$a_5 + a_1 = s5$

Let us apply the rule to a simple problem (not discussed by Aryabhata which can be formulated symbolically as

$x_1 + x_2 = 13$

$x_2 + x_3 = 14$

$x_1 + x_3 = 15$

Applying Aryabhata's rule, the solution would be based on the rule for deriving the sum of all three unknown quantities as

$$x_1 + x_2 + x_3 = \frac{13+14+15}{3-1} = 21$$

This allows us to determine the value of the quantities by subtracting the partial sums from the total with the solution (7, 6, 8).

From the ninth century we find a derived version of the previous problem in Hindu sources. Mahavira gives an elaborate description of the rule in the Ganitasarasamgraha (GSS, stanza 233-5, Padmavathamma 2000, 357-9).

The rule for arriving at the value of the money contents of a purse which when added to what is on hand with each of certain persons becomes a specified multiple of the sum of what is on hand with the others:

The quantities obtained by adding one to each of the specified multiple numbers in the problem and then multiplying these sums with each other, giving up in each case the sum relating to the particular specified multiple, are to be reduced to their lowest terms by the removal of common factors. These reduced quantities are then to be added. Thereafter the square root of this resulting sum is to be obtained, from which one is [to be subsequently subtracted. Then the reduced quantities referred to above are to be multiplied by this square root as diminished by one. Then these are to be separately subtracted from the sum of those same reduced quantities. Thus the moneys on hand with each of the several persons are arrived at.

These quantities measuring the moneys on hand have to be added to one another, excluding from the addition in each case the value of the money on the hand of one of the persons and the several sums so obtained are to be written down separately. These are then to be respectively multiplied by the specified multiple quantities mentioned above; from the several products so obtained the already found out values of the moneys on hand are to be separately subtracted. Then the same value of the money in the purse is obtained separately in relation to each of the several moneys on hand. The introductory sentence states that the rule is to be used for determining the value of a purse. The rule is followed by a number of problems that begin as "Four men saw on their way a purse containing money" (ibid. stanza 2451 2, 367). This is the earliest instance, in our investigation of the sources, in which the popular problem of men finding a purse is discussed. While problems with the same structure and numerical values have been formulated before, the context of men finding a purse seems to have originated in India before 850 AD. Formulations with the purse turn up in Arabic algebra with al- Karkhi's Fakhri (c. 1050) and in the Miftah al - muamalat of al Tabari (c. 1075). Fibonacci has many variations of it in the Liber Abbaci (1202) and after that it becomes the most common problem in western arithmetic until the later sixteenth century. For an understanding of the rule, let us look at its application to a given problem (GSS, stanza 236-7, pp. 360). Three merchants saw [dropped] on the way a purse [containing money]. One [of them] said [to the others], "If I secure this purse, I shall become twice as rich as both of you with your moneys on hand". Then the second of them said, "I shall become three times as rich". Then the other, the third, said, "I shall become five times as rich". What is the value of the money in the purse, as also the money on hand with each of the three merchants?

We can represent the problem in symbolic equations

$x + p = 2 (y + z)$

$y + p = 3 (x + z)$

$z + p = 5 (x + y)$

Let us apply the recipe of Mahavira to this problem, step by step. By "adding one to [each of the specified] multiple numbers" we have 3, 4 and 6. "Multiplying these sums with each other" we get 72. This has to be "reduced to their lowest terms by the removal of common factors". This least common multiple is 12. The reduced quantities are then 4, 3 and 2 respectively. Adding all three together gives 9, from this the square root is 3. Then the reduced quantities "are to be multiplied by the square root as diminished by one", which is 2. This leads to 8, 6 and 4. The money in hand for each of the merchants now is the difference of these values with the sum of the reduced quantities, being 9. The solution thus is 1, 3 and 5. The rest of the rule is an elaborate way to derive the value of the purse. Using the values in any one of the equations immediately leads to 15 for the value of the purse. Mahavira provides no explanation or derivation of the rule. For a mathematical argument for the validity of the rules see Heeffer (2007a).

We know almost nothing about Thymaridas of Paros, but he is supposed to have lived between 400 and 350 BC (Tannery 1887, 385-6). The only extant witness is Iamblichus, in his comments on the Introduction to Arithmetic by Nichomachus of Gerasa. The best known source for The Bloom of Thymaridas is Heath's classic on Greek mathematics. Heath (1921, 94) does not formulate the rule, he only observes that "the rule is very obscurely worded" and writes out the equations. The text from Iamblichus was first published in Holland with a Latin translation by Samuel Tennulius (1668) from the Paris manuscript BNF Gr. 2093. A critical edition, based on multiple manuscripts was published by Pistelli (1884). Nesselmann (1842, 233) quotes the Greek text and the Latin translation from Tennulius, who translated the method as florida sententia. We give here the own literal translation from Pistelli (1884, 62). From this we are also acquainted with the method of the Epanthema, passed down to us by Thymaridas. Indeed, when a given quantity divides into determined and unknown parts, and the unknown quantity is paired with each of the others, so will the sum of these pairs, diminished by the sum [of all the quantities] be equal to the unknown quantity in case of three quantities. With four quantities it will be half of it, with five it will be a third, with six, a fourth and so on. The rule is not as obscure as considered by Heath. Let us extract the basic elements of the rule, and compare these with the version of Aryabhata:

1. The rule applies to any number of quantities, as does Aryabhata's.
2. The sum is given in the problem. The rule is described as the division of a known quantity in determined and undetermined parts. In Aryabhat.a's rule the sum is what is looked for.
3. The partial sums are the sums of the pairs of the unknown part with each of the known quantities. In Aryabhata's rule the partial sums include all the numbers except one.

In short, this rule is different from Aryabhata's in two important aspects. Its intention is to find one unknown part of a determined quantity. Aryabhata's rule is meant for finding the sum of numbers of which the partial sums of all minus one is given. Even in the case of three numbers, when the partial sums are the same, the rules have different applications. To make it clear to the modern era, here is a symbolic version in the general case

$$x + a_1 + a_2 + \ldots\ldots.. a_{n-1} = s$$
$$x + a_1 = s_1$$
$$x + a_2 = s_2$$
$$.$$
$$.$$
$$.$$
$$x + a_{n-1} = s_{n-1}$$

$$x = \frac{\sum_{i=1}^{n-1} s_i - s}{n-2}$$

Conclusion

The humanist project of reviving ancient Greek science and mathematics played a crucial role in the creation of an identity for the European intellectual tradition. While Greek mathematics was hardly known or practiced before the fifteenth century, humanist mathematicians identified themselves with this tradition. When Regiomontanus declared that algebra was invented by Diophantus, humanist writers rejected the Arabic roots of algebra, though it was practiced and turned into an independent tradition for two centuries in Italian cities such as Florence and Sienna. The newly created identity of mathematics descending from ancient Greek thinkers blurred historical perception. When Indian algebra and arithmetic was introduced into Europe, the leading historians of the nineteenth-century could only see its alleged relation with Greek mathematics. The Bloom of Thymaridas is an excellent illustration of distorted historical investigation. Apparently nineteenth-century historians found it difficult to accept that mathematics is a human intellectual activity endeavoured across cultures within societies that needed and supported the achievements of mathematical practice. A true history of mathematics should take into account contributions of all origins.

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