International Journal of Engineering, Science and Mathematics
Vol.6Issue 4, August 2017,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

AN INVENTORY MODEL WITH VARIABLE DEMAND FOR NON-INSTANTANEOUS DETERIORATING ITEMS UNDER PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT

In order to encourage the retailers to buy more, in today's competitive business transaction, the suppliers normally permit his/her retailers a delay in payment. In this permissible delay period, the retailer is allowed to postpone payment for the products bought without paying any interest. In realistic circumstances, the problem of determining the optimal replenishment policy for non-instantaneous deteriorating items with multivariate demand is important which is discussed in this study. Finally, a numerical example is presented to demonstrate the developed model analysis.

Keywords: Inventory; Inflation, Multivariate demand;Non-instantaneous deterioration; Ordering cost; Purchasing cost and Permissible delay in payments.

INTRODUCTION

In the current age the presentation of the products/items in large quantities in the supermarkets attracts more and more customers and generates the advanced demand. Thus the effect of inflation and multivariate demand cannot be ignored for getting the optimal inventory policy. It has been study that the most researchers on the inventory models do not assumed the trade credit with the inflation simultaneously. Trade credit and inflation play an important role in the optimal order policy inventory model and influences the demand of certain products/items. Buzacott (1975) established an EOQ model with inflation condition. During the long ago sometimes, a lot of number of researchers have discussed inventory models with the permissible delay in payment condition. Goyal (1985) was the first to discussed an EOQ constant demand inventory model with the condition of permissible delay in payments. Later, Aggarwal and Jaggi (1995) expanded Goyal's inventory model to consider an inventory model using fix deterioration rate.

Gupta and Vrat (1986) first proposed a mathematical model system which is optimize the total inventory cost and considering the stock-dependent demand for utilization environment. Liao et al. (2001) proposed an inventory model with stock dependent consumption rate under the permissible delay in payment conditions. Jaggi *et. al.*, (2006) discussed an inventory model with constant deteriorating rate and the demand rate increase exponentially due to inflation. Pal and Ghosh (2006) discussed an inventory model with quantity dependent permissible delay in payment condition and shortage. Soni and Shah (2008) designed the mathematical model considering stock-dependent demand with optimal ordering policy under the progressive payment scheme environment. Singh and Malik (2009) discussed a mathematical model for inventory management system under the stock dependent demand rate for non-instantaneous decaying items with two storage capacity system. Singh and Malik (2009) proposed a two warehouses inventory model with inflation induced demand under the credit period condition. Chang *et. al.,* (2010) discussed an improved model with optimal replenishment policy under the stock-dependent demand rate.

Singh and Malik (2010) investigated an inventory system with variables parameters. Singh *et. al.,* (2011) presented an inventory model with multivariate demand rate. Khanra *et. al.,* (2011) developed an inventory model for decaying item with the quadratic demand under the permissible delay in payment condition. Sharma *et. al.,* (2013), Gupta et al. (2013), Singh *et. al.,* (2013) and Malik *et.al.,* (2016, 2017) developed the inventory model for non-instantaneous deteriorating items. Singh et al. (2014) discussed a stock-dependent demand model under the Permissible delay in Payment and inflation. Vashistha *et. al.,* (2015) developed an optimum inventory system with maximum life time items in which demand rate is considered as a function of price and stock- dependent. Kumar *et. al.,* (2016)proposed a mathematical model considering variable holding cost under the stock-dependent demand rate with non-instantaneous decaying products. Recently, Kumar *et. al.,* (2017) demonstrate an optimal inventory model assuming with variable holding and variable sales revenue cost.

NOTATIONS AND ASSUMPTIONS

For the developed inventory model, we use the following notations and assumptions:

- The demand rate is $D(t) = a_1 + a_2 t + a_3 I_{inv}(t)$, where $(a_1, a_2, a_3) \geq 0$ all are positive constants and $I_{inv}(t)$ is the inventory level at time *t*.
- Shortages are not acceptable and lead time is zero.
- The retailer can accumulate revenue and earn interest after customers pay for the amount of purchasing cost to the retailer in anticipation of the completion of the permissible delay in payment period offered by the supplier.

α	deterioration rate
C_O	ordering cost per order
C_H	Inventory holding cost per unit time
C_D	Deteriorating cost per unit
C_P	Purchasing cost per unit
C_S	Sales revenue cost per unit
r	Discount rate, representing the time value of money
i	Inflation rate

R	Net discount rate of inflation; R = *r-i*
M	Permissible delay in payment offered by supplier in months
I_p	Interest charges per \$ per month
I_e	Interest earned per \$ in stocks per month
TVP	Total profit of the proposed inventory system per unit time

MATHEMATICAL MODEL

During the time interval [0, t_1], the inventory level I_{nv1} (t) decreases due to multivariate demand rate. The inventory level drops to zero due to demand and the deterioration in the items during the interval is [t_1, T]. $I_{nv1}(t)$ indicates the inventory level at time $0 \le t \le t_1$ in which the item has no deterioration, $I_{nv2}(t)$ is the inventory level at time $t_1 \le t \le T (= t_1 + t_2)$ in which the product has deterioration. Thus, the proposed inventory level at any time t can be represented by the following differential equations:

$$\frac{dI_{nv1}(t)}{dt} = -\left(a_1 + a_2 t + a_3 I_{nv1}(t)\right) \qquad 0 \le t \le t_1 \qquad \ldots. (1)$$

$$\frac{dI_{nv2}(t)}{dt} + \alpha I_{nv2}(t) = -\left(a_1 + a_2 t + a_3 I_{nv2}(t)\right) \qquad t_1 \le t \le T \qquad \ldots. (2)$$

Showing the boundary conditions $I_{nv1}(0) = I_W$, $I_{nv2}(T) = 0$ respectively, solving above equations (1) and (2), we get

$$I_{nv1}(t) = (I_W + f_1)e^{-a_3 t} - f_1 - \frac{a_2}{a_3} t, \qquad 0 \le t \le t_1 \qquad \ldots. (3)$$

$$I_{nv2}(t) = (f_2 - t f_3) + e^{(a_3+\alpha)(T-t)}(T f_3 - f_2), \quad t_1 \le t \le T \ldots\ldots\ldots\ldots\ldots\ldots. (4)$$

Where

$$f_1 = \left(\frac{a_1 a_3 - a_2}{a_3^2}\right), \quad f_2 = \left(\frac{a_2 - a_1(a_3 + \alpha)}{(a_3 + \alpha)^2}\right), \quad f_3 = \left(\frac{a_2}{a_3 + \alpha}\right)$$

Considering continuity of $I_{nv}(t)$ at t=t_1, it follows from Equations (3) and (4) that $I_{nv1}(t_1) = I_{nv2}(t_1)$

$$\Rightarrow I_W = \left(\frac{a_2}{a_3} t_1 + f_1 + f_2 - t_1 f_3 \right) e^{a_3 t_1} - f_1 - e^{(a_3+\alpha)T} \left(f_2 - T f_3 \right) \qquad \ldots(5)$$

Inventory ordering cost (IOC) is = C_O ….(6)

Inventory holding cost (IHC) is

$$IHC = C_H \left(\int_0^{t_1} e^{-Rt} I_{rv1}(t)dt + \int_{t_1}^{T} e^{-Rt} I_{rv2}(t)dt \right) \qquad \ldots\ldots(7)$$

Inventory deterioration cost (IDC) is

$$IDC = C_D \int_{t_1}^{T} \alpha e^{-Rt} I_{rv2}(t)dt \qquad \ldots(8)$$

Inventory purchasing cost (IPC) is

$$IPC = C_P \times I_W = C_P \left[\left(\frac{a_2}{a_3} t_1 + f_1 + f_2 - t_1 f_3 \right) e^{a_3 t_1} - f_1 - e^{(a_3+\alpha)T} \left(f_2 - T f_3 \right) \right] \qquad \ldots(9)$$

Inventory sales revenue cost (ISRC) is

$$ISRC = C_S \int_0^{T} e^{-Rt} \left(a_1 + a_2 t + a_3 I_{rv}(t) \right) dt$$

In this paper we have considered permissible delay in payment in two periods: (based on the length of T and M)

Case-1: $t_1 \leq M \leq T$, in this case, the interest payable is

$$IP_1 = C_P I_p \int_M^T I_{mv2}(t)\, dt \qquad (11)$$

The interest earned is

$$IE_1 = C_S I_e \left[\int_0^{t_1} t.(a_1 + a_2 t + a_3 I_{mv1}(t))dt + \int_{t_1}^{M} t.(a_1 + a_2 t + a_3 I_{mv2}(t))\, dt \right] \quad \ldots.(12)$$

Total profit TVP_1 per unit time is

$$TVP_1 = \frac{1}{T}\left[ISRC - IOC - IHC - IDC - IPC - IP_1 + IE_1\right] \quad \ldots (13)$$

The total profit TVP_1 is maximum if

$$\frac{dTVP_1}{dt_2} = 0 \text{ and } \frac{d^2TVP_1}{dt_2^2} < 0 \quad \ldots.(14)$$

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Case-I1: M≥T, in this case, the no interest charges are paid for the items, i.e.,

$$IP_2 = 0 \qquad \text{…. (15)}$$

The interest earned is

$$IE_2 = C_S I_e \left[\int_0^{t_1} t.(a_1 + a_2 t + a_3 I_{rw1}(t))dt + \int_{t_1}^{M} t.(a_1 + a_2 t + a_3 I_{rw2}(t))\,dt + D(t)T(M - T) \right] \qquad \text{…. (16)}$$

The total profit TVP_2 is maximum if

$$TVP_2 = \frac{1}{T}\left[ISRC - IOC - IHC - IDC - IPC - IP_2 + IE_2\right] \qquad \text{…(17)}$$

The necessary and sufficient conditions for TVP_2 (total present value of profit per unit time) is maximum

$$\text{if } \frac{dTVP_2}{dt_2} = 0 \text{ and } \frac{d^2TVP_2}{dt_2^2} < 0 \qquad \text{…. (18)}$$

NUMERICAL EXAMPLE

To illustrate the above results, we may consider the following examples:

Ex.1. Consider C_O=1100, C_H= 0.50, C_D= 0.08, C_P= 100, C_S= 200, I_p= 0.15, I_e=0.10, a_1= 150, a_2= 0.3, a_3=0.1, α=0.03, and R=0.02. **Case-1** assume M=0.6 month, and **Case-2** assume M=0.8 month.After solving (13) we observe that the profit(TVP_1) is maximum when t_1^*=1/2 month, t_2^*=1.1103 month, TVP_1^*=13405.16 and optimal order quantity is I_W^*=286.6135. After solving (17) we observe that the profit (TVP_2) is maximum when t_1^*=1/2 month, t_2^*=0.276376 month, TVP_2^*=13192.8166 and optimal order quantity is I_W^*=140.1165.

The following graphs (Fig. 1.1 and 1.2) show the relation between total profit ($TVP_1^*$and TVP_2^* with respect to t_1* and t_2*)

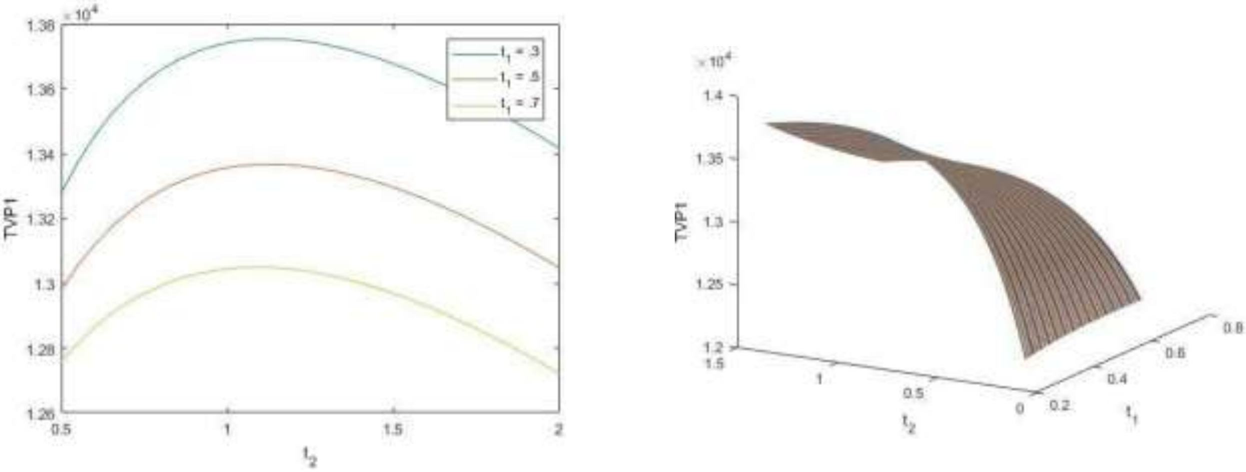

Fig 1.1: 2D and 3D view of Total Profit TVP_1* v/s t *and t *values

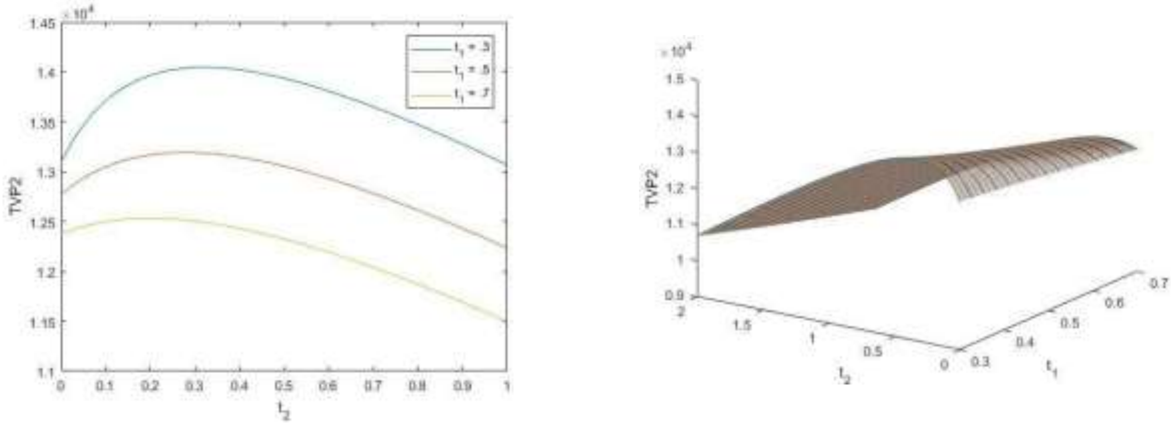

Fig. 1.2: 2D and 3D view of Total Profit TVP_2 v/s $t_2^*$and t_1^*values

CONCLUSION

The purpose of this article is to study the methods and modelling of the inventory system of natural environment (variable demand, credit and deterioration) on the final results of conducting smooth any business organization. This paper deals with the idea of an inventory control system beside the non-instantaneous deteriorating items with the inflation and trade credit conditions. Numerical example has been considered in this proposed inventory model to illustrate the significant features of the results. Further the possible future research for the study of an inventory model for variable demands and deterioration rates, production dependent model, partial backlogging, two warehouses etc.

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