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ON SOME THEOREMS ASSOCIATED WITH A SYSTEM OF SIMULTANEOUS DIFFERENTIAL EQUATIONS CONSTRUCTION OF BOUNDARY CONDITION VECTORS

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Abstract: In this paper some theorems associated with a system of simultaneous differential equations construction of Boundary Condition Vectors have been proved.

Keywords: Sim. Diff. Equ^n, Boundary Conditions

1. **Introduction:** We consider the following system of differential equations:

$$\left.\begin{aligned} u'' + pu + qv + rw &= \lambda u \\ qu - v'' + rv + sw &= \lambda v \\ ru + sv + iw + pw &= \lambda w \\ nu + qv + rw - ix &= \eta x \end{aligned}\right\}$$

Where u, v, w, x are functions p, q, r, s are real valued conditions functions of t, l, o, μ, v, η are parameters which may be real or complex, $t\varepsilon\,[a, b], i = \sqrt{-1}$, and dashes denote derivatives w.r.t. t.

2. **Theorem:** The system (1.1) of differential equations yields (admits) a unique solution

$$\theta(t) = (u\ v\ w\ x)^t(t)$$

satisfying the initial conditions

$$\left.\begin{aligned} u^{(s)}(\alpha) &= A_s \\ A^{(s)}(\alpha) &= B_s \\ w(\alpha) &= C_0 \\ x(\alpha) &= D_0 \end{aligned}\right| \qquad (1.2)$$

where $A_s, B_s(s = 0,1), C_0, D_0$ are arbitrary constants (real or complex) not all vanishing simultaneously. T denotes transpose (s) denotes sih derivatives w.r.t. t and $\alpha\varepsilon[a, b]$.

Proof: The system of differential equations (1.1) and set of initial conditions (1.2) may be alternatively written as:

$$\left.\begin{aligned} u'' &= -lv - mw - nx + \lambda u \\ v'' &= lu + pw + qx - \mu v \\ w' &= imu + ipy + irx - ivw \\ x' &= -inu - iqv - irw + i\eta x \end{aligned}\right| \quad (1.3)$$

$$\left(u(\alpha), u(\alpha), v(\alpha), v'(\alpha), w(\alpha), x(\alpha)\right)$$

$$= (A_0 A_1 B_0 B_1 C_0 D_0) \quad (1.4)$$

Further for a vector V let V^T denote the transpose of V and

$$V^T = (u\,u'v\,v'w\,x)$$

where dashes denote derivatives w.r.t. t; then (1.3) and (1.4) have their respective equivalent forms as:

$$V'(t) = F(t)V(T)$$

and

$$V(\alpha) = (A_0 A_1 B_0 B_1 C_0 D_0)^T \quad (1.5)$$

where

$$F(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \lambda & 0 & -\lambda & 0 & -m & -n \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & -\mu & 0 & p & q \\ im & 0 & ip & 0 & -iv & ir \\ -in & 0 & iq & 0 & -ir & i\eta \end{bmatrix}$$

Since V and F both are complex hence we can write them as:

$$\left.\begin{aligned} &V = V_1 + iV_2 \\ &and \\ &F = F_1 + iF_2 \end{aligned}\right| \quad (1.6)$$

Where V_1, V_2 and F_1, F_2 are real matrices.

With the help fo (1.6) we get from (1.5)

$$w'(t) = \begin{bmatrix} F_1 & F_2 \\ F_2 & F_1 \end{bmatrix} w(t)$$

where

$$w = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad W_0 = \begin{bmatrix} V_1(\alpha) \\ V_2(\alpha) \end{bmatrix} \quad (1.7)$$

By Picard's theorem (Chapter 1 and 2 of ref 1. The expressions (1.7) yields a unique solution $\phi(t) = \left(u(t)v(t)w(t)t.x(t)\right)^T$ depending analytically on λ.

This proves the theorem.

3. **Construction of Boundary Condition Vectors:**

We use the symbol

$$\phi(\alpha/x) = \left(u\left(\frac{\alpha}{x}\right)v\left(\frac{\alpha}{x}\right)^T (\alpha, x \in [a,b])\right)$$

To denote a solution of (1.1) satisfying a set of conditions of the form

$$\left(u^{(r)}\left(\frac{\alpha}{x}\right)\right)_{x=a} = u^{(r)}(\alpha/x) = A_r(r = 0,1,2)$$

and

$$\left(v^{(s)}\left(\frac{\alpha}{x}\right)\right)_{x=\alpha} = v^{(s)}(\alpha/x) = B_s(s = 0,1) \qquad (3.1)$$

where (r) denotes rth detivative w.r.t. x.

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