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Fuzzy Sets, Fuzzy S-Open and S-Closed Mappings

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1.Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper [1]. This concept provides a natural foundation for treating mathematically the fuzzy phenomena, which exist pervasively in our real world, and for building new branches of fuzzy mathematics.In the area of Fuzzy Topology, introduced by Chang[2],much attention has been paid to generalize the basic concepts of General Topology in fuzzy setting and thus a modern theory of Fuzzy Topology has been developed.

In recent years, Fuzzy Topology has been found to be very useful in solving many practical problems. Du et al. [3] fuzzified the very successful 9-intersection Egenhofer model [4, 5] for depicting topological relations in Geographic Information Systems (GIS) query. In [6, 7], El Naschie showed that the notion of Fuzzy Topology might be relevant to quantum particle physics and quantum gravity in connection with string theory and e∞ theory. Tang [8] used a slightly changed version of Chang's fuzzy topological space to model spatial objects for GIS databases and Structured Query Language (SQL) for GIS.

Levine [9] introduced the concepts of semi-open sets and semicontinuous mappings in topological spaces. Interestingly, his work found applications in the field of Digital Topology [10]. For example, it was found that the digital line is a T1/2-space [11], which is a weaker separation axiom based upon semi-open sets. Fuzzy Digital Topology [12] was introduced by A. Rosenfeld, which demonstrated the need for the fuzzification of weaker forms of notions ofClassical Topology. Azad [13] carried out this fuzzification in 1981, and presented some general properties of fuzzy spaces. Several properties of fuzzy semi-open (resp., fuzzy semi-closed), fuzzy regular open (resp., closed) sets have been discussed. More-over he defined fuzzy semicontinuous (resp., semi-open and semi-closed) functions and studied the properties of fuzzy semicontinuous function in product related spaces. Finally, he defined and characterized fuzzy almost continuous mappings. For related subsequent work in this direction, we refer to [14–27].

In this paper, our aim is to further contribute to the study of fuzzy semi-open and fuzzy semi-closed sets defined by Yalvac [26] by establishing several important fundamental identities and inequalities supported by counterexamples. Cameron and Woods [28] introduced the concepts of s-continuous mappings and s-open mappings. They investigated the properties of these mappings and their relationships to properties of semi-open sets. Khan and Ahmad [29] further worked on the characterizations and properties of s-continuous, s-open and s-closed mappings. We fuzzify the findings of [28, 29]. We define fuzzy s-open and fuzzy s-closed mappings and establish some interesting characterizations of these mappings.

2.Preliminaries

In order to make this paper self-contained, we briefly recall certain definitions and results; for those not described, we refer to [1, 2, 13, 26].

Let X ={ x}be a space of points (objects), with a generic element x. A fuzzy set λ in X is characterized by membership function λ(x) fromX to the unit interval [0,1].

The symbol Φ denotes the empty fuzzy set defined as μΦ(x) = 0, for all x ∈ X. For X, the membership function is defined as μX(x)=1, for all x ∈X.

Definition 1 (see [2]). Let f : X → Y be a mapping. Let β be a fuzzy set in Y with membership function β(y). Then the inverse of β, written asf−1(β), is a fuzzy set in X whose membership function is defined by

X. (1) $$^{-1}(\beta)(x) = \beta(f(x)), \quad \forall x \in$$

Conversely, let λ be a fuzzy set in X with membership function λ(x). The image of λ, written asf(λ), is a fuzzy set in Y whose membership function is given by

$$f(\lambda)(y) = \begin{cases} \sup_{x\in f^{-1}(y)}\{\lambda(x)\}, & \text{if } f^{-1}(y) \text{ is nonempty,} \\ 0, & \text{otherwise} \end{cases}$$

Definition 2 (see [2]). A fuzzy topology is a family τ of fuzzy sets in X, which satisfies the following conditions:

(1) Φ,X ∈τ;

(2) If λ,μ∈τ, then λ∧μ∈τ;

(3) If λi ∈τ for each i∈I, then ∨iλi ∈τ.

τ is called a fuzzy topology for X, and the pair (X, τ) is an fts. Every member of τ is called τ-open fuzzy set (or simply an open fuzzy set). A fuzzy set is τ-closed if and only if its complement is τ-open. As in general topology, the indiscrete fuzzy topology contains only Φ and X, while the discrete fuzzy topology contains all fuzzy sets.

3.Fuzzy Semi-Open and Fuzzy Semi-Closed Sets

Definition 3 (see [13]). Let λ be a fuzzy set in an fts (X, τ). Then λ is called a fuzzy semi-open set of X, if there exists a $\nu \in \tau$ such that $\nu \le \lambda \le Cl\nu$. A fuzzy set μ is fuzzy semi closed if and only if its complement μc is fuzzy semi-open. The class of all fuzzy semi-open (resp., fuzzy semi-closed) sets in X is denoted by FSO(X) (resp., FSC(X)).

Definition 4 (see [26]). Let λ be a fuzzy set in an fts X. Then semi-closure (briefly sCl) and semi-interior (briefly sInt) of λ are defined as

$$\begin{aligned} \mathrm{sCl}\lambda &= \wedge\{\beta \mid \lambda \le \beta, \beta \text{ fuzzy semi-closed}\} \\ sint\lambda &= \vee\{\beta \mid \beta \le \lambda, \beta \text{ fuzzy semi-open}\} \end{aligned} \quad (3)$$

and are called the fuzzy semi-closure of λ and fuzzy semi interior of λ, respectively.

It is immediate that

(1) $\mathrm{sCl}\lambda \ge \lambda$ and $\mathrm{sInt}\lambda \le \lambda$;

(2) $\lambda \le \mu \Rightarrow \mathrm{sCl}\lambda \le \mathrm{sCl}\mu$, $\mathrm{sInt}\lambda \le \mathrm{sInt}\mu$.

It is known [13] that a fuzzy set λ in an fts X is

(1) fuzzy semi-closed if and only if $\mathrm{IntCl}\lambda \le \lambda$ (resp., $\mathrm{sCl}\lambda=\lambda$);

(2) fuzzy semi-open if and only if $\lambda \le \mathrm{ClInt}\lambda$ (resp., $\mathrm{sInt}\lambda=\lambda$).

The following are characterizations of fuzzy semi-closed sets, the proof of Theorem 1 is straightforward.

Theorem 1. A fuzzy set λ is fuzzy semi-closed if and only if there exists a fuzzy closed set ψ such that $\mathrm{Int}\psi \le \lambda \le \psi$.

Theorem2. Forafuzzyset λ inanfts X, λ isfuzzysemi-closed if and only if $\mathrm{sIntsCl}\lambda \le \lambda$.

Proof. (⇒) If λ is a fuzzy semi-closed set, that is, λ = sClλ, then sIntsClλ = sIntλ ≤ λ.

(⇐) Suppose sInt (sClλ) ≤ λ. Since sCl λ is fuzzy semi closed, so there exists a closed set ψ such that Int ψ ≤ sClλ ≤ ψ. Thus Intψ ≤ sIntsClλ ≤ λ ≤ sClλ ≤ ψ or Intψ ≤ λ ≤ ψ. Hence λ is fuzzy semi-closed and λ=sClλ.

Remark 1. It is easily seen that

(1) if λ is fuzzy semi-open (resp., fuzzy semi-closed), then Intλ, sInt λ, sCl λ, and Clλ are fuzzy semi-open (resp., fuzzy semi-closed);

(2) a nonvoid nowhere dense fuzzy set λ is fuzzy semi closed and not fuzzy semi-open.

The converse of Remark 1(2) is, in general, not true as is shown by following.

Example 1. Let X = {a, b, c}be a set and I = {0,.3,.5,.7,1} be the lattice of membership grades for fuzzy sets in X. Let μ = {a.7, b0, c1}, ν = {a.7, b. 5, c.3}, and ω = {a.5, b.5, c.5} be fuzzy sets on X, and τ the fuzzy topology generated by μ, ν, and ω. Then τ = {Φ, μ, ν, ω, {a.5, b0, c.5}, {a.5, b.5, c.3}, {a.7, b0, c.3},{a.7, b0, c.5},{a.7,b.5,c.5},{a.7,b.5,c1},X}. Calculations give that fuzzy set ω is both fuzzy semi-closed and fuzzy semi-open but IntClω=ω.

Theorem3. For fuzzy sets λ and μ in an fts X, one has

(1) sInt (λ ∨ μ) ≥ sInt λ ∨ sInt μ;

(2) sInt (λ ∧ μ) = sInt λ ∧ sIntμ;

(3) sCl (λ ∨ μ) = sClλ ∨ sClμ;

(4) sCl (λ ∧ μ) ≤ sClλ ∧ sClμ.

Proof. (1) sInt λ and sInt μ are both fuzzy semi-open. λ ≤ λ ∨ μ, μ ≤ λ ∨ μ imply sIntλ ≤ sInt(λ ∨ μ) and sIntμ ≤ sInt(λ∨μ). Combining, sIntλ∨sIntμ≤sInt(λ∨μ) or

$$sint(\lambda \vee \mu) \geq sint\lambda \vee sint\mu \quad (4)$$

(2) λ∧μ≤λ and λ∧μ≤μ imply sInt(λ∧μ)≤sIntλ∧ sIntμ. Conversely sIntλ ≤ λ and sIntμ ≤ μ imply sIntλ∧ sIntμ ≤ λ ∧ μ and sIntλ ∧ sIntμ is fuzzy semi open. But sInt (λ∧μ) is the largest fuzzy semi open set such that sInt(λ∧ μ)≤λ∧μ, hence sIntλ ∧ sIntμ ≤ sInt(λ ∧ μ). This gives the equality.

(3) and (4) follow easily from (2).

The inequalities (1) and (4) of Theorem 3, are in general irreversible, as is shown by following.

Example 2. Let X = {a, b} be a set and I = {0, .5, 1} be the lattice of membership grades for fuzzy sets on X. Let μ = {a.5, b1}, ν = {a.0, b1}, and ω = {a1, b.5}be fuzzy sets on X, and τ the fuzzy topology generated by μ, ν, and ω. Then

$$\tau = \{\Phi,\mu,\nu,\omega,\{a.5,b.5\},\{a0,b.5\},X\} \quad . (5)$$

We choose fuzzy sets α = {a.5, b_0}, β = {a_0, b.5}, γ = {a.5, b1}and δ = {a1, b.5}. Then calculations give that

sInt {α∨β} ={a.5,b.5} <{a0,b.5}=sIntα∨sIntβ,

sClγ∧sClδ ={ a1,b.5}/ <{a.5,b.5}=sClγ∧δ————————————.(6)

Itisknown[30]thatforafuzzyset λ inanfts X,wehave

(1) ClsClλ=Clλ;

(2) IntClλ≤sClλ.

We use this and Theorem 1, and prove the following.

Theorem 4. Let λ be a fuzzy set in an fts X. Then one has the following:

(1) sClsClλ=sClλ(sIntsIntλ=sIntλ);

(2) sIntsClλ≥IntClλ;

(3) IntsIntsClλ=IntClλ;

(4) sIntsClλ≤IntClλ∨λ.

Proof. (1) By the fact that sClλ is fuzzy semi-closed and that λ is fuzzy semi-closed if and only if sClλ = λ, it follows immediately.

(2) Since, IntClλ ≤ sClλ, so that IntClλ ≤ sIntsClλ, since all fuzzy open sets are fuzzy semi-open.

(3) By (2), IntClλ ≤ sIntsClλ, so that IntClλ ≤ IntsIntsClλ. Also sClλ ≤ Clλ, so that sIntsClλ ≤ Clλ and IntsIntsClλ≤IntClλ. Consequently, we get (3).

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(4) Now IntClλ ≤ IntClλ∨λ ≤ Clλ so by Theorem 1, IntClλ∨λ is fuzzy semi-closed. Also λ ≤ IntClλ∨λ. Since sClλ is the smallest fuzzy semi-closed set with λ ≤ sClλ, therefore, sClλ ≤ IntClλ ∨ λ. Which implies sIntsClλ ≤ sClλ≤IntClλ∨λ or sIntsClλ≤IntClλ∨λ.

The inequalites (2)and(4)of Theorem 4may,ingeneral, not be reversible as is shown in the following.

Example 3. Let X ={ a,b,c}be a set and I ={0,.3,.5,.7,1} be the lattice of membership grades for fuzzy sets on X. Let μ ={ a.5,b1,c.3}, ν ={ a0,b.5,c1}, andω ={ $a_{.3}$,$b_{.7}$,$c_{.5}$}

be fuzzy sets on X, andτ the fuzzy topology generated by μ, ν, andω. Then τ ={Φ,μ,ν,ω,{a_0,$b_{.5}$,$c_{.5}$},{a_0,$b_{.5}$,$c_{.3}$},{a.3,b.7,c.3},{a.5,b1,c.5},{a.3,b.7,c1},{a.5,b1,c1},X}.

We choose fuzzy sets λ ={ a.7,b.5,c.5} and ψ = {a0,b0,c.5}, then calculations give that

sIntsClλ={ a.7,b.5,c.5}/<{a0,b.5,c.5}=IntClλ; (7)

IntClψ∨ψ ={ a0,b0,c.5}/ <{a0,b0,c0}=sIntsClψ.

Theorem5. For any fuzzy set λ in an fts X, we have

(1) (sIntλ)c =sCl(λc),

(2) (sClλ)c =sInt(λc).

Proof. (1)

$(sInt\lambda)^c = 1 - sInt\lambda$

$= 1 - \vee\{\mu | \mu \in FSO(X) \text{ and } \mu \le \lambda\}$

$= \wedge\{1 - \mu | \mu \in FSO(X) \text{ and } \mu \le \lambda\}$ (8)

$= \wedge\{\psi \,|\psi^c \in FSO(X) \text{ and } \psi \ge \lambda c\}$,

where $\psi = 1 - \mu$

$= sCl(\lambda c)$.

(2) Similar to (1).

4.FuzzyS-OpenandFuzzyS-ClosedMappings

First, we define

Definition 5. A functionf : X → Y is said to be fuzzy sopen (resp., fuzzy s-closed) if the image of every fuzzy semiopen (resp., fuzzy semi-closed) set is fuzzy open (resp., fuzzy closed).

Obviously a fuzzy s-open function is fuzzy open.

Definition 6 (see [31]). A fuzzy point e is called a boundary point of a fuzzy set λ if and only if e ∈ Clλ ∧ Clλc. The union of all the boundary points of λ is called a boundary of λ, denoted by Bdλ. It is clear that

$$Bd\lambda = Cl\lambda \wedge Cl\lambda c. \quad (9)$$

Next, we define

Definition 7. Semiboundary (briefly sBd) of a fuzzy set λ in an fts X is defined as

$$sBd\lambda = sCl\lambda \wedge sCl\lambda^{c}. \quad (10)$$

In the following, we characterize fuzzy s-open mappings in terms of sInt, sCl, and sBd.

Theorem 6. For a function f : X → Y, a fuzzy set α in an fts X and a fuzzy set β in an fts Y, then the following are equivalent:

(1) *f* is fuzzy s-open;

(2) $f(sInt\alpha) \leq Int\ f(\alpha)$;

(3) $sInt\ f^{-1}(\beta) \leq f^{-1}(Int\beta)$;

(4) $f^{-1}(Cl\beta) \leq sCl(f^{-1}(\beta))$;

(5) $f^{-1}(Bd(\beta)) \leq sBd(f^{-1}(\beta))$.

Proof. (1) ⇒ (2) obviously $f(sInt\alpha) \leq f(\alpha)$. *f* is fuzzy s-open gives $f(sInt\alpha)$that is fuzzy open in Y. But $Intf(\alpha)$ is the largest fuzzy open set such that $Int\ f(\alpha) \leq f(\alpha)$. Therefore $f(sInt\alpha) \leq Int\ f(\alpha)$, for any fuzzy set α in X. This gives (2).

(2) ⇒ (3) For any fuzzy set β in Y, $f{-}1(\beta) = \alpha$ is a fuzzy set in X. Then by (2)

$$f\left(sintf^{-1}(\beta)\right) \leq Int\left(f\left(f^{-1}(\beta)\right) \leq Int(\beta)\right.$$

$$\text{or } f\left(sintf^{-1}(\beta)\right) \leq Int\beta \text{ or } sintf^{-1}(\beta)$$

$$f^{-1}f\left(sintf^{-1}(\beta)\right) \leq f^{-1}(Int\beta) \text{ or sint } f^{-1}(\beta) \leq f^{-1}(Int\beta)$$

This gives (3).

(3)⇒ (4) By (3), we have

$$sintf^{-1}(\beta) \leq f^{-1}(Int\beta)$$

$$\left(f^{-1}(Int\beta)\right)^c \leq \left(sintf^{-1}(\beta)\right)^c \qquad (12)$$

$$= sCl\left(f^{-1}(\beta)\right)^c \qquad \text{(by Theorem 5(1))}$$

or $\left(f^{-1}(\text{Int}\,\beta)\right)^c \leq \text{sCl}\left(f^{-1}(\beta)\right)^c$ or $f^{-1}(\text{Cl}\beta^c) \leq \text{sCl} f^{-1}(\beta^c)$ or $f^{-1}(\text{Cl}\psi) \leq \text{sCl} f^{-1}(\psi)$ where $\psi = \beta^c$, a fuzzy set in Y. This gives (4). (4)⇒ (5) For a fuzzy set β in Y, $\text{Bd}\beta = \text{Cl}\beta \wedge \text{Cl}(\beta^c)$ is a fuzzy closed set in Y. Now $f^{-1}(\text{Bd}\beta) = f^{-1}(\text{Cl}\beta) \wedge f^{-1}(\text{Cl}\beta^c)$. Using (4) we have

$$f^{-1}(\text{Bd}\beta) \leq \text{sCl}\left(f^{-1}(\beta) \wedge \text{sCl}\left(f^{-1}\left(\beta^c\right)\right)\right) \qquad (13)$$

Or

$$f^{-1}(\text{Bd}\beta) \leq \text{sCl} f^{-1}(\beta) \wedge sCl\left(f^{-1}(\beta)\right)^c = sBdf^{-1}(\beta) \qquad (14)$$

This gives (5).

In the following, we give characterizations of fuzzy sclosed mappings as follows.

Theorem 7. A function*f* : X → Y is fuzzy s-closed i*f* and only i*f* Cl f(λ)≤*f*(sClλ), for each fuzzy set λ in an fts X. Proof. (⇒) Obviouslyf(λ) ≤ f(sClλ). f(sClλ) is fuzzy closed, since f is fuzzy s-closed. But Cl f(λ) is the smallest fuzzy closed set with f(λ) ≤ Cl f(λ). Therefore Cl f(λ) ≤ f(sClλ).

(⇐) Let$\lambda \in FSC(X)$. We show that f(λ) is fuzzy closed. By hypothesis, $Cl\, f(\lambda) \leq f(sCl\lambda) = f(\lambda)$ or $Clf(\lambda) \leq f(\lambda)$. This proves that f(λ) is fuzzy closed.

Theorem8. If a function $f : X \to Y$ is fuzzy s-closed then for each fuzzy set β in an fts Y and each fuzzy semi-open set μ in an fts X with $\mu \geq f-1(\beta)$, there exists a fuzzy open set ν in Y with $\nu \geq \beta$ such that $f-1(\nu) \leq \mu$.

Proof. Letμbeanarbitraryfuzzysemi-opensetinX with$\mu \geq f-1(\beta)$, where β is a fuzzy set in Y. Clearly (f(μc))c =ν (say) is fuzzy open in Y. Since $f-1(\beta) \leq \mu$, then straightforward calculations give that $\beta \leq \nu$. Moreover, we have

$$f^{-1}(\nu) = f^{-1}(f(\mu c))^c$$

$$= (f^{-1}f(\mu c))^c \leq \mu$$

or $f^{-1}(\nu) \leq \mu$. (15)

Theorem 9. Let $f : X \to Y$ be a surjective function from an fts X to an fts Y. If for each fuzzy set β in Y and each fuzzy semi-opensetμinX with$\mu \geq f^{-1}(\beta)$,thereexistsafuzzyopen set ν in Y with $\nu \geq \beta$ such that $f-1(\nu) \leq \mu$, then f is s-closed. Proof. Let ψ be an arbitrary fuzzy semi-closed set in X and $y \in (f(\psi))c$. Then

$$F^{-1}(y) \leq f^{-1}(f(\psi))^c = (f^{-1}f(\psi))^c \leq \psi^c (16)$$

or $f^{-1}(y) \leq \psi^c$. Since ψ^c is fuzzy semi-open, therefore there exists a fuzzy open set ν_y with $y \in \nu_y$ such that $f^{-1}(\nu_y) \leq \psi^c$. Since f is surjective, we have $y \in \nu_y \leq (f(\psi))^c$. Thus $(f(\psi))^c = \vee\{ \nu_y \mid y \in (f(\psi))^c\}$is fuzzy open in Y or $f(\psi)$ is fuzzy closed in Y. This proves that *f* is s-closed.

Combining Theorems 8 and 9, we have the following.

Theorem 10. A surjective function $f : X \to Y$ is fuzzy sclosed if and only if for each fuzzy set β in Y and each fuzzy semi-opensetμ InX with$\mu \geq f^{-1}(\beta)$,thereexistsafuzzyopen set ν in Y with $\nu \geq \beta$ such that $f-1(\nu) \leq \mu$.

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