Some Aspects of Electrogravitation in the Reissner-Nordstrom Space-Time

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Abstract—Considering circular trajectories and using Shirokov's technique based on geodesic deviation, frequencies of θ- vibrations and r or (ϕ) – vibrations of a particle moving along circular geodesic in the Reissner-Nordstrom space-time are obtained. Furthermore, effect of charge on the vibrations of the test particles instead of helping the matter to curve the space-time more, decurves the space-time which means that the nature of gravitational field due to the matter and charge matter may be of different type. Our investigation conclude that, the vibrations of the test particle in the $(\xi^1 - \xi^3)$-plane are elliptical and along ξ^2 harmonically rectilinear.

Keywords—Shirokov's technique, geodesic deviation Effect of charge, θ- vibrations, r or (ϕ) – vibrations, Theory of Relativity, gravitational field.

1 Introduction

EINSTEIN'S theory of gravitation, though it has formal beauty and mathematical elegance has been accepted as physical theory on the basis of experimental verifications of the observable effects referred as test of general theory of relativity GTR. Of all the tests "one new effect of Einstein's theory of gravitation", known as Shirokov's effect is a new test.

According to Shirokov (1973), if a test particle moving in the Schwarzschild field along a circular orbit is put to vibrate then the periods of θ- vibrations and r or (ϕ) – vibrations are

$$T_\theta = \frac{2\pi}{\Omega} = T_0\left(1 - \frac{3m}{2r}\right) \text{and}$$

$$T_r\left(or\ T_\phi\right) = \frac{2\pi}{\omega} = T_0\left(1 + \frac{3m}{2r}\right)$$

resulting in

$$\Delta T_{Schl'd} = T_\theta - T_r\left(or\ T_\phi\right) = T_0(-3m/r)$$

(where $T_0 = 2\pi\,(r^3/m)^{1/2}$ is the Newtonian period of a test particle in the circular orbit of radius r) showing that θ- vibrations lie behind r or (ϕ) – vibrations by $(3m/r)T_0$ as a new effect of Einstein's theory of gravitation.

Further in his study by deriving 4-deviation vector ξ^i, shown that in the $(\xi^1 - \xi^3)$-plane the vibrations of the test particle are elliptical and along ξ^2 harmonically rectilinear.

In this paper we have applied Shirokov's technique to study this effect in the electrogravitational field. Our investigations conclude that, the vibrations of the test particle in the $(\xi^1 - \xi^3)$-plane are elliptical and along ξ^2 harmonically rectilinear.

In section 2, following Howes[2], expressions for the frequencies of vibrations of the test particle are derived in the R-N field. In section 3, 4-deviation vector ξ^i is obtained and thereby effect of the charge on the vibrating system is discussed.

2 Frequencies Of Vibrations

In the general theory of relativity, the equation of deviation from the geodesic [1] is

$$\frac{d^2\xi^i}{ds^2} + 2\,\Gamma^i_{jk} u^j \frac{d\xi^k}{ds} + \frac{\partial \Gamma^i_{jk}}{\partial x^l} u^j u^k \xi^l = 0\,, \tag{1}$$

where ξ^i is the infinitesimal 4-vector giving the deviation from the basic geodesic, $u^i = dx^i/ds$ is the 4-velocity vector tangential to the basic geodesic and Γ^i_{jk} are Christoffel symbols defined as

$$\Gamma^i_{jk} = \frac{1}{2} g^{li}\left(\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}\right).$$

We suppose that the basic geodesic is a circular trajectory with radius $r =$ constant in the plane $\theta = \pi/2$ in the Reissner-Nordstrom (R-N) field,

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2) + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) dt^2 \tag{2}$$

where$r = x^1, \;\; \theta = x^2, \;\; \phi = x^3, \;\; t = x^4$.

For the field (2), metric tensors are

$$g_{11} = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}, \quad g_{22} = -r^2, \quad g_{33} = -r^2 sin^2\theta\,,$$

$$g_{44} = \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right), \qquad g_{ij} = 0 \text{ for } i \neq j \tag{3}$$

and the non-vanishing components of the Christoffel symbols are

$$\Gamma^1_{11} = -\left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1},$$

$$\Gamma^1_{22} = -r\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right), \; \Gamma^1_{33} = -r\, sin^2\theta\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right),$$

$$\Gamma^1_{44} = \left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right), \Gamma^2_{21} = \frac{1}{r} = \Gamma^3_{13}\,,$$

$$\Gamma^2_{33} = -sin\,\theta\, cos\,\theta\,, \qquad \Gamma^3_{23} = cot\,\theta\,,$$

$$\Gamma^4_{41} = \left(\frac{m}{r^2} - \frac{e^2}{r^3}\right)\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}. \tag{4}$$

Following Howes[2], if the basic geodesics are circular in the axisymmetric stationary field, θ-disturbances are independent of r, ϕ, and t-perturbations.
Therefore for $i = 2$, equation (1) assumes the form

$$\frac{d^2\xi^2}{ds^2} + \frac{\partial \Gamma^2_{jk}}{\partial x^2} u^j u^k \xi^2 = 0 (\,j, k = 1, 2, 3, 4\,) \tag{5}$$

If we suppose that

$$\xi^2 = \xi^2_0 e^{i\,\Omega\, s} \tag{6}$$

(ξ^2_0 is the amplitude of θ-vibrations) then from (5), we obtain

$$\Omega^2 = \Gamma^2_{jk\,,2} u^j u^k\,, \tag{7}$$

where comma in the Christoffel symbol denotes the partial differentiation and Ω is the frequency of θ-vibrations.
For $i = 1, 3, 4$, from (1), we get

$$\frac{d^2\xi^1}{ds^2} + 2\,\Gamma^1_{j3} u^j \frac{d\xi^3}{ds} + 2\,\Gamma^1_{j4} u^j \frac{d\xi^4}{ds} + \Gamma^1_{jk\,,1} u^j u^k \xi^1 = 0,$$

$$\frac{d^2\xi^3}{ds^2} + 2\,\Gamma^3_{j1} \frac{d\xi^1}{ds} u^j = 0\,, \quad \text{and}$$

$$\frac{d^2\xi^4}{ds^2} + 2\,\Gamma^4_{j1} \frac{d\xi^1}{ds} u^j = 0 \tag{8}$$

Further, if we suppose that

$$\xi^j = \xi_0^j e^{i\omega s}, \quad (j = 1, 3, 4) \tag{9}$$

(ξ_0^j is the amplitude of r, ϕ and t-vibrations), then from (8), we get

$$(\Gamma_{jk,1}^1 u^j u^k - \omega^2)\xi_0^1 + 2i\omega\Gamma_{j3}^1 u^j \xi_0^3 + 2i\omega\Gamma_{j4}^1 u^j \xi_0^4 = 0,$$

$$2i\omega\Gamma_{j1}^3 u^j \xi_0^1 - \omega^2\xi_0^3 = 0, \quad \text{and}$$

$$2i\omega\Gamma_{j1}^4 u^j \xi_0^1 - \omega^2\xi_0^4 = 0 \tag{10}$$

whereω is the frequency of r, ϕ and t-vibrations, all the Christoffel symbols and their derivatives are evaluated at $\theta = \pi/2$.
For non-trivial solution of (10), we equate the determinant of coefficients to zero and obtain

$$\omega^2 = u^j u^k \Gamma_{jk,1}^1 - 4u^j u^k \Gamma_{j1}^3 \Gamma_{k3}^1 - 4u^j u^k \Gamma_{j1}^4 \Gamma_{k4}^1$$

or

$$\omega^2 = (\Gamma_{33,1}^1 - 4\Gamma_{31}^3\Gamma_{33}^1)(u^3)^2 + (\Gamma_{44,1}^1 - 4\Gamma_{14}^4\Gamma_{44}^1)(u^4)^2 \tag{11}$$

To determine u^3, consider geodesic equation

$$\frac{du^i}{ds} + \Gamma_{jk}^i u^j u^k = 0, \ (i, j, k = 1, 2, 3, 4) \tag{12}$$

in the Einstein's theory of gravitation.
For circular orbits in the equatorial plane, from (12) we find that

$$\frac{dt}{d\phi} = \frac{u^4}{u^3} = \left(\frac{-\Gamma_{33}^1}{\Gamma_{44}^1}\right)^{\frac{1}{2}}, \tag{13}$$

which provides the angular velocity of the test particle as seen from the infinity.
Using (4) in (13), we get

$$(u^4)^2 = \frac{r^2}{\left(\frac{m}{r} - \frac{e^2}{r^2}\right)}(u^3)^2. \tag{14}$$

For equatorial circular orbit in the field (2) using (13) we get

$$(u^3)^2 = \left(\frac{m}{r^3}\right)\left(1 - \frac{e^2}{mr}\right)\left(1 - \frac{3m}{r} + \frac{2e^2}{r^2}\right)^{-1} \tag{15}$$

The corresponding frequencies of θ-vibrations and r (or ϕ)-vibrations in (7) and (11) simplify to

$$\Omega^2 = (u^3)^2 \tag{16}$$

&

$$\omega^2 = (u^3)^2\left\{1 - \frac{6m}{r} + \frac{3e^2}{r^2} + \frac{e^2}{mr} + \frac{e^4}{m^2r^2} + O(\eta)^{2\frac{1}{2}}\right\} \tag{17}$$

respectively, in which $\frac{m}{r} = \frac{e}{r} = O(\eta)$, η is small.

3 Periods Of Vibrations

The periods of θ- vibrations and r or (ϕ) – vibrations are

$$T_\theta = \frac{2\pi}{\Omega} = T_0\left(1 - \frac{3m}{2r} - \frac{9m^2}{8r^2} + \frac{e^2}{2mr} + \frac{e^2}{3r^2} + \frac{3e^4}{8m^2r^2} + O(\eta)^{2\frac{1}{2}}\right) \tag{18}$$

and

$$T_r(or\ T_\phi) = \frac{2\pi}{\omega} = T_0\left(1 + \frac{3m}{2r} + \frac{63m^2}{8r^2} - \frac{7e^2}{2r^2} + O(\eta)^{2\frac{1}{2}}\right) \tag{19}$$

where$T_0 = 2\pi\,(r^3/m)^{1/2}$ is the Newtonian period of a test particle in the circular orbit of radius r. The difference ΔT_{RN} between the periods of θ- vibrations and r or (ϕ) – vibrations is

$$\Delta T_{RN} = \left(-\frac{3m}{r} + \frac{e^2}{2mr}\right)T_0 \tag{20}$$

to the 1 ½ order approximation.

For $e = 0$, (20) reduces to

$$\Delta T_{Schl'd} = -\frac{3\,m}{r}\,T_0 - T_r\ , \qquad (21)$$

the result obtained by Shirokov (1973) as a new effect of
Einstein's theory of gravitation.

From (20) and (21) we find a relation between shift in periods of θ- vibrations and r or(ϕ) – vibrations in R-N field and Schwarzschild field as

$$\Delta T_{RN} = \Delta T_{Schl'd}\left(1 - \frac{e^2}{6\,m^2}\right) \qquad (22)$$

which is obtained by KalpanaPawar& G. D. Rathod[5].

4 Infinitesimal Deviation Four-Vector

Using (4), (9) in (8) and then simplifying, we get

$$\xi_0^3 = 2\left(1 + \frac{3m}{r} - \frac{e^2}{2m\,r}\right) e^{i\frac{\pi}{2}}\xi_0^1\ , \qquad (23)$$

$$\xi_0^4 = 2\left(\frac{m}{r}\right)^{\frac{1}{2}} e^{i\frac{\pi}{2}}\xi_0^1 \qquad (24)$$

Now the equations (6) and (9) in real quantities can be expressed as

$$\xi^1 = \xi_0^1 \sin\omega\, s\ , \qquad \xi^2 = \xi_0^2 \sin\Omega\, s\ ,$$

$$\xi^3 = 2\left(1 + \frac{3m}{r} - \frac{e^2}{2m\,r}\right)\xi_0^1 \cos\omega\, s \text{ and } \qquad \xi^4 = 2\left(\frac{m}{r}\right)^{\frac{1}{2}}\xi_0^1 \cos\omega\, s\ . \qquad (25)$$

4 Conclusion

The relation (22) between shift in periods of θ- vibrations and r or(ϕ) – vibrations in R-N field and Schwarzschild field is analogous to the relation,

$$\delta\phi_{RN} = \delta\phi_{Schl'd}\left(1 - \frac{e^2}{6\,m^2}\right)$$

between perihelic shift in R-N field and Schwarzschild field obtained by H. J. Treder, H. H. V. Borzeszkowski, A. Van Der Merwe, W. Y. Yourgrau[3].

According to G. D. Rathod and T. M. Karade[4], $\delta\phi_{RN} < \delta\phi_{Schl'd}$ shows that charge on the gravitating particle instead of helping the matter to curve the space-time more, decurves the space-time, which means that the nature of the gravitational fields due to the matter and charged matter may be of different type.

In our case from (22), we find a similar relation, $\Delta T_{RN} < \Delta T_{Schl'd}$,which supports the conclusion of G. D. Rathod and T. M. Karade[4], and K. Pawar and G. D. Rathod[5].

From (25) it is clear that, in the $(\xi^1 - \xi^3)$-plane the vibrations of the test particle are elliptical and along ξ^2 harmonically rectilinear as in the Schl'd field.

For $e = 0$, one can recover all the results of Shirokov (1973). Moreover, we observe that minor axis of elliptical path in the $(\xi^1 - \xi^3)$-plane gets contracted as an effect of charge.

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