International Journal of Engineering, Science and Mathematics
Vol. 9 Issue 12, December 2020,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

SOME QUEUEING CHARACTERIZATION TECHNIQUES

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ABSTRACT

Queueing theory is concerned with mathematical modeling and analysis of system that provide service to random demands. A queueing model is an abstract description of such a system. A queueing model represents the system's physical configuration and arrangement of the servers, which provide service to the customers and stochastic (i.e. probabilistic or statistical) nature of the demand, by specifying the variability in the arrival process and in the service process. The art of applied queueing theory is to construct a model that is simple enough so that it yields to mathematical analysis, yet contains sufficient detail so that its performance measures reflect the behavior of the real system. The analytical queueing models can be solved by using several classical techniques. Here we'll provide a brief outline of some techniques used for analyzing the queueing characterization for real time systems.

Introduction: Queueing theory was developed to provide models for efficient management of the stochastic systems that provide service to randomly arising demands of varying nature with variety of service mechanisms. An analytical approach is used to obtain the explicit expression, but, numerical methods provide an alternative approach using inherent structure to analyze the queueing system. Model is first formulated mathematically as a continuous time markov process with discrete states resulting the birth-death Chapman-Kolomogorov differential equations. Some of them are briefly discussed here.

a. Direct Method

This is a numerical method, which yields the exact results in a finite number of arithmetic operations. But in practice, it produces approximate results due to round off errors. Gauss elimination method is the examples of direct method.

b. Iterative Methods

The iterative method for the solution of a system of linear equations governing steady state queueing models is frequently used. Iterative method starts with an initial approximation and proceeds with appropriate algorithm to obtain successively better approximation. By using this method we preserve the sparsely parameter matrix. Successive convergence to the desired solution is also provided by an iterative algorithm. A good initial estimate can speed up the computation considerably. The iterations can be terminated by imposing a pre-specified tolerance level. Round-off errors are also avoided by this method because the parameter matrix is not altered in this method.

The power method, Gauss Seidel and Successive Over Relaxation (SOR) techniques, are some iterative methods for solving the system of linear equations describing the queueing problems.

c. Supplementary Variable Techniques

In the supplementary variable technique a non Markovian process in continuous time is made Markovian by the inclusion of one or more supplementary variables. If we know how long a customer had been in service at any time, the distribution of the time to the next state transition, together with the probabilities of the next possible states, would be known. Thus, if we include in the state the time elapsed since the current customer being served (if any) entered service, we would indeed have the Markov property at all times and hence a continuous time Markov chain. To illustrate this point we cite the M/G/1 queueing system studied by Cox (1955). In M/G/1, since the service time distribution is general, let the state of the system be defined by a pair of variables, the number N(t) in the system at epoch t and the elapsed service time X(t) of the customer who is undergoing service. Then we study the bivariate Markovian process {N(t), X(t)} in order to obtain results for the non-Markovian process {N(t)}. Thus Cox defines $P_n(v, t)$ to be the joint probability and p.d.f. of n, the number of customer in the system, including the one being served, and v the elapsed service time of the customer in the service. The inclusion of a single supplementary variable 'v' makes the process Markovian in continuous time. The supplementary variable technique has been employed to analyze queueing models developed in chapters 6, 7 and 8. The service times are i.i.d. r.v's whose p.d.f. is

$$b(v) = \eta(v)\exp[-\int_0^v \eta(x)dx]$$

with mean 1/μ and corresponding density function is given by

$$\exp[-\int_0^v \eta(x)dx] = 1 - B(v).$$

d. Matrix Geometric Method

This procedure can be used when the rate matrix has a particular block lower (or upper) Hessenberg structure. A special case of this general structure, which is commonly found in queueing systems, is the case where the rate matrix has the following block tri-diagonal structure

$$Q = \begin{pmatrix} A' & B' & & & & \\ C' & A & B & & & \\ & C & A & B & & \\ & & C & A & B & \\ & & & . & . & . & \\ & & & & . & . & . \end{pmatrix} \qquad (1)$$

where each letter represents a block sub-matrix.

Let x be the vector of steady state probabilities associated with Q, i.e. xQ=0 and xe=1, where e is a vector of 1's. We partition x with the blocks of matrix Q. For instance, lets have an example in which sub vector x_0 has 3 components and sub vector x_i, i=1,2,…. , has 4 components. (x_0, x_1, x_2,…, x_i,…)Q=0 can be written out as

$$x_0 A' + x_1 C' = 0$$
$$x_0 B' + x_1 A + x_2 C = 0$$
$$x_i B + x_{i+1} A + x_{i+2} C = 0,\ i = 1,2....$$

By manipulating the above equations, x_i, i=1,2… can be expressed in terms of x_1 as follows:

$$x_i = x_1 R^{i-1},\ i = 1,2.... \quad (2)$$

where R is the minimal non-negative solution of the matrix equation

$$B + RA + R^2 C = 0 \quad (3)$$

In certain cases, it is possible to obtain a closed form expression for R. In general, however, R is obtained recursively by rewriting (1) as

$$R_{n+1} = -B(A^{-1}) - (R_n^2 C)(A^{-1})$$

And setting R_0=0. Sub vectors x_0 and x_1 can be obtained from equations

$$\begin{cases} x_0 A' + x_1 C' = 0 \\ x_0 B' + x_1 (A + RC) = 0 \end{cases} \quad (4)$$

and normalizing condition. In matrix geometric procedure, the normalizing equation can be used in conjunction with (1.8) in order to solve for x_0 and x_1. Alternatively, it can be handled as follows. One of the elements of x_0 or x_1 is assigned the value 1. Then, all other probabilities are calculated accordingly using (1) and (4). Because of this, it is possible that some probabilities may have a value greater than 1. Finally, each of these probabilities is divided by the sum of the entire probabilities so that they all add up to 1.

e. Probability Generating Function Techniques

The probability generating functions (PGF) approach is a convenient tool that simplifies computations involving integer valued, discrete random variables. The probability generating function is a powerful tool to tackle queueing problems and to provide solution of difference equations. By using the generating function the discrete sequence of numbers (probabilities) is transformed in the function of dummy variable.

Given a non negative integer valued discrete random variable X with $P(X=k)=p_k$, we define the PGF of X as:

$$G_X(z) = \sum_{i=0}^{\infty} p_i z^i, \quad |z| < 1$$

$G_X(z)$ is also known as the z- transform of X. It may easily verify that $G_X(1) = 1 = \sum_{i=0}^{\infty} p_i$.

Probability generating function technique can be employed to obtain explicit results in case where it may not otherwise be possible to obtain easily.

f. Runge Kutta Method

The Runge-Kutta algorithm can be used to solve a set of differential equations numerically.

Consider a set of differential equations that can be written as

$$X' = F(t, X)\ ;\ X(a) = S$$

where $X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ and F is the vector. We can write

$$X_i' = f_n(t, X_1, X_2, \ldots\ldots, X_n)$$

for i=1,2,3,…,n and $X_i(a)=S_i$, i=1,2,3,…,n.

and let

$$X = [X_1, X_2, X_3, \ldots\ldots, X_n]^T$$

$$F = [f_1, f_2, f_3, \ldots\ldots, f_n]^T$$

$$X' = [X_1', X_2', X_3', \ldots\ldots, X_n']^T$$

$$S = [S_1, S_2, S_3, \ldots\ldots, S_n]^T$$

R-K method gives the algorithm to solve differential equations as

$$X(t+h) = X + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where

$$k_1 = F(t, X)$$

$$k_2 = F(t + \frac{h}{2}, X + \frac{h}{2}k_1)$$

$$k_3 = F(t + \frac{h}{2}, X + \frac{h}{2}k_2)$$

$$k_4 = F(t + \frac{h}{2}, X + hk_3)$$

The Runge-Kutta algorithm is known as well behaved and very accurate for a wide range of congestion problems and can be implemented by using the "ode 45" function in software MATLAB.

g. Diffusion Approximation

When random variables defining the system change continuously instead of discretely, the system is characterized by a probability density function that satisfies a second order partial differential equation called Focker-Planck equation or diffusion equation. For the diffusion approximation, we propose that the arrival process and the departure process are both to be approximated by continuous random processes that at time t are normally distributed.