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PULSE PROPAGATION DISPERSIVE MEDIUM

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Abstract –

We first consider the mathematical theory of boundary initial value problems for Maxwell' s equations. An extensive discussion of ID- problems, Then, with the objective to investigate electromagnetic pulse propagation in dispersive media, we analyse how to translate electromagnetic processes which take place in a bounded domain of space

– time into a boundary – initial value problem of Maxwell' s equations, still focusing on 10 – problems.

1. Introduction –

To analyse electromagnetic wave propagation in any medium most people stick to station' s philosophy [1] " A pulse or signal of any specified initial form may be constructed by superposition of harmonic wave trains of infinite length and duration" A curious statement, germane to the idea that any function has a fierier transform act that could be jeopardized by the behavior of digital signals generated in the modern technology of communication. We depart here from this philosophy.

When one considers the propagation of electromagnetic fields in a bounded region Ω of space, during a finite interval of time (O,T), as it is the case for instance, for any computer modulation of electromagnetic processed, one has to deal with a boundary initial value (B-I) problem, requiring initial data at t = 0 and another set of data on the boundary Ω. In order to make these problems well posed for Maxwell's equations, these data have to satisfy some constraints not always easy to formulate mathematically, that is why we investigate these constraints for ID- Problems where calculations are less intricate Then, an important question that we also examine in this case, is how to translate an electromagnetic physical processed into a correct B-I problem of Maxwell's equations. We assume that the material in the region Ω is made of a Maxwell Hopkinson (M-H for briet) dielectric which is the simplest dispersive medium that one can imagine.

The objective of this paper is not to present a general theory of B-I problems in electromagnetic but only to show on simple examples the pitfalls to be avoided.

Then, the paper is organized as follows, sec 2 is devoted to the mathematical background of B-I problems for Maxwell's equations and to the constraints to be imposed on boundary to be imposed on boundary and initial data in the frame of 1D-Problems, Sec. 3 is concerned with the translation of electromagnetic propagation in Ωx[O,T] into a B-I problem of Maxwell's equations still in the case of 1D-Problems conclusive comments are given in sec. 4.

2. B-I problems for Maxwell's equations

With the light velocity unity, Maxwell's equations in a no conducting medium $\{X = (x, y, z)\}$

$$\text{Cur IE (x,t)} = -\partial + B\ (x, t)$$

$$\text{Cur IH (x,t)} = \partial + D\ (x, t)$$

$$\text{divE}\ (x, t) = \text{diVB (x,t)} = 0$$

EM pulse propagation in dispersive meia

Become with B=μH, leaving aside the divergence equations Cur IE (x,t) = -

$\mu\partial + H(x, t)$

$$\text{Cur IH (x,t)} = \partial + D\ (x, t) \qquad \text{(1a)}$$

From which we get the following equation where is the laplacian operator

$$\Delta\ E\ (x, t) - \ \partial^2 + D(x, t) = \ 0 \qquad \text{(2)}$$ Now,

M-H medium is characterized by the constitute relation [2]

$$D(x, t) = \epsilon\ E(x, t) + \int_{o}^{t} \emptyset\ (t - \tau)E\ (x, \tau)\tau \qquad [3]$$

Where ϵ is a positive constant and ∅ (t) , t > o , a monotonically decreasing function of + continuous for o < t < ∞ , note that along this paper E is zero in the M-H medium for t < o. The displacement field D satisfies an Integra differential equation [3] which seems to have been largely unnoticed.

$$\varepsilon\mu\partial t^2(x, t) = \Delta D(x, t) + \int_{o}^{I} \Psi\ (t - \tau)\Delta\ D\ (x, \tau)d\tau\ (\theta)$$

In which $\Psi(t)$ is given by the iterative series whose convergence is discussed in [9]

$$\Psi\ (t) = \sum_{n-1}^{\infty} (-1)^n\ \Psi_n(t)$$

$$\Psi_1(t) = \in^{-1} \Psi(t)$$

$$\Psi(t) = \int_o^t \Psi_1 (t-\tau)\Psi_{n-1}(\tau), n \geq 2$$

To look for the solutions of eq. (2) with D given by (3) , we use the laplaco transform [5]

$$f(s) = \int_o^{\infty} exi\,(-st)F\,(t)dt, Re\ s > o, t \geq o \qquad [5]$$

So that with evident notations an taking into account the property of the laplace transform to change a convolution product into an ordinary product, the constitutive relation (3)

Becomes

$$d(x,s) = \in e(x, s) + \emptyset\,(s)e[x, s] \qquad [6]$$

Then assuming the initial conditions

$$D\,(x,o) = \in E\,(x, o) = F(x)$$

$$x\ (x, o) = \in\ \partial_t E(x, o) = \ \pi\,(X6) \qquad [7]$$

And using the la place transform way of tacking derivatives the integrodifferential equations changes into

$$\Delta\, d\,(x, s) + (s)d(x,s) - n_2[s^2 d\,(x, s) - \delta\,F\,(x) - \pi\,(x)] = o \qquad n_o^2 = \in \mu \qquad [8]$$

While eq. (2) becomes

$$\Delta(x, \delta) - \mu\,[\delta^2 d(x, \delta) - \delta F(x) - G\,(x)] = 0 \qquad [9]$$

And substituting (6) into (9) gives

$$\Delta(x, \delta) - \delta^2 n(s) e(x, \delta) = v\,(x, s) \qquad [10]$$

$$n^2 s = [E + \emptyset(s)], \quad v\,(x, s) = -\mu\,(\delta f(x) \; + \; U\,(x)) \qquad [10a]$$

But for physical reasons and to satisfy causality, the partial differential equation (10) has to be hyperbolic and it has been provide (3) that this condition is fullfille if $\emptyset$ (δ) / i (s) with degree q > degree p so that according to (100) n^2 (s) is the quotient of two polynomials with the same degree and

$$\lim_{\delta = \infty} n^2(s) = n^2_0 = \in \mu \qquad [11]$$

For instance, in a M-H dielectric [2], $\emptyset(s) = \sum^n_0 (s + an)^{-1}$ where α_n, a_n, are positive contents depending on the constitutive material of the dielectric and the Davis criterion is satisfied.

Now , as said in the introduction one is interested in the solutions of eq.
(10) in some region Ω of space so that one has still to supply data on its boundary or and the specification of these data can become quite involve
[1] to pase physical problems in a convenient manner. The intal conditions impose some constraints on the solutions of (10) that also intervene to limit the set of possible boundary data require to define a well posed problem. To make calculations easier without losing the essentials, we discuss these constraints in the next two sections on the solutions of the ID- Partial differential equation .

$$\partial^2_z\,(z, s) - s^2 n^2\,(\delta)\,(z, s) = v\,(z, s)$$

$$(z, s) = -\mu\,[sf(z) + G(2)] \qquad [12]$$

As a simple example, the solution of the B-I problem for the ID-Scalar wave equation

$\partial^2_z \Psi$ (z, t) − ∂^2_t Ψ (z, t) = o, z t ≥ o 1 [13]

With the initial and boundary conditions Ψ (o,t) =

h(t) + ≥ o , Ψ (z,o) = f (z)

$\partial_1 \Psi(z, o) = y(2)$ 126

Where f,g,h are given functions with continuous second derivatives and moreover

h(o)=f(o), $h^1(o) = g(o), h^{II}(o) = f^{II}(o)$ [120]

has continuous second derivatives even on the characteristics line z – t = o

of course, once known e (x,s), one has still to perform an inverse laplace transform to obtain E (x,t) either by using tables [6], by analytical calculations in the complex δ - plane [7]

or by numerical computation [10]

constraints imposed by initial conditions with n written for n(s) , one proves in Appendix A that the general solution of (12) is

$$e\,(z, s) = a\,(s)\,e^{-snz} + b\,(s)\,e^{snz-}\,({}^{1}\!/_{2}{}^{sn})\int_{\infty}^{o} du e^{sn}$$

$$V\,(z + u, s) - ({}^{1}\!/_{2}{}^{sn})\int_{o}^{x} du e^{snv}\,v\,(2tu, s) \qquad \text{(B)}$$

In which a (s) and b (s) are two arbitrary functions.

But, according to the Abel – tuber theorem [s]. limits sf (s) = lim F (t) when both limits s=100 exist, t = 0 so the solutions (B) must satisfy the conditions obtained from (7)

Lim se (2,5) = E (z,o) = $\in^{-1}, F$ (2) (14a)

S= 00

Lim s [se (z,s)-v (2,5)] = ∂, E (z, o) = $\in^{-1}$ 07 (z) (14b)

S=100

Which implies in particular

Lim sa (s)^{snz} = 0, lim sb $(s)e^{sm}$ = 0 (15)

And assuming that a (s) and b (s) are not exponentially increasing or decreasing at infinity, we ge

a(s) = 0 , zco b(s) = 0 z > 0 (15a)

Taking into account (15) , the relations (14a) , (14b) become

Lim

S= ∞ $\{-\left(\frac{1}{2}m\right)\int_{\infty}^{0} du\, e^{snu} G\ (2+4) + \left(\mu s/20\right)$

$-\infty$

$\int_{0}^{\infty} du\, e^{snu}\ G\ (2+4)\} = \in^{-1}\ G\ (2)$ (16b)

These relations are checked in Appendix B with the help of (II)

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