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Covering approximation space for Lattice

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Abstract

In 1981 Prof.Pawlak introduced Rough set theory. It covers the areas of research in artificial intelligence, knowledge representation system, imprecision, vagueness and others. All vague concepts is substituted by a pair of precise concepts- one is the lower approximation and other is the upper approximation of the vague concept. The lower approximation consists of all objects which definitely belong to the concept and the upper approximation contains all objects which probably belong to the concept. The difference between these two approximations constitutes the boundary region of the vague concept. Lattice concept was introduced by Peirce and Schroder. Structure of Lattices are simple since the basic concepts of the theory include only least upper bounds , greatest lower bounds and the order relation. Now it plays an important role in many disciplines of engineering and computer science .

Here we applied the rough set theory to the lattice theory and a concept introduced here known as lattice rough, which deals with the problems of uncertainties in our day to day life.

Keywords:

Rough Set,
Lattice,
Lattice Rough,
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1. Introduction

The lattice structure appeared in the middle of 19th century in the form of Boolean algebra due to George Boole 1854. Boolean algebra deals with operations on logical values and binary variables 0 and 1, that is 1(True) or 0(False). Now a days Lattice theory has a great importance in the field of artificial intelligence[6], knowledge representation system, engineering, computer science etc. The theory of rough set, proposed by Prof. Z. Pawlak([4, 5]) is a powerful mechanism to handling and processing the incomplete information in expert system. The main advantage of this set theory is that it does not requre any preliminary or additional information about data. The rough set approach is based on knowledge with granular structure, which caused by the situation when objects of interest cannot be distinguished and they may appear to be identical. The indiscernibilty relation thus generated is the mathematical basis of Pawlak's Rough set theory.

Here our aim is to introduce roughness to lattice theory through the covering approximation space. At first we define a covering on a lattice L and a mapping $R : L \rightarrow L$. Then we find covering lattice approximation space and using the lower approximation and upper approximation we define lattice rough in L.

2. Lattice

A relation R on a set S is called a partial order([1], [2]) if it satisfies the following conditions

1. Reflexive: dRd for all d ∈ S,

2. Antisymmetric: dRe and eRd⇒ d = e,

3. Transitive: dRe and eRc ⇒ dRc for all c, d, e ∈ S.

A set S together with a partial order relation R is called a partially ordered set or a Poset. It is denoted by (S,R). The relation R is often denoted by the symbol □. Hence a Poset is denoted by (S, □).

Definition 2.1. A lattice L is a partially ordered set where two elements a and b of L have $a \vee b$ and $a \wedge b$ are in L.

Definition 2.2. A sublattice $(L_1, \vee, \wedge)$ of a lattice $(L, \vee, \wedge)$ is defined on a nonempty subset L_1 of L which satisfied the condition, $x, y \in L_1$ implies that $x \vee y$, $x \wedge y \in L_1$.

Proposition 2.3. If x, y, z are the elements of lattice L,

1. $x \vee x = x$ and $x \wedge x = x$
2. $x \vee y = y \vee x$ and $x \wedge y = y \wedge x$
3. $x \vee (y \vee z) = (x \vee y) \vee z$ and $x \wedge (y \wedge z) = (x \wedge y) \wedge z$.
4. $x \vee (x \wedge y) = x$ and $x \wedge (x \vee y) = x$.

Definition 2.4. If 1 and 0 are the greatest and least element of a lattice L, then x is known as the complemented element of L if there exists an element $x^{/} \in L$ such that $x \wedge x^{/} = 0$ and $x \vee x^{/} = 1$. where $x^{/}$ is known a complement of x.

3. Rough Set

A set of objects U , known as universal set, be a finite one. Let A be a set of attributes. Then we denote Knowledge Representation System (an information system) by (U, A). For a function, $e : U \rightarrow Ve$, Ve is called a set of values of attribute e, for each e ∈ A.[3].

Let R be an equivalence relation (knowledge) defined on the non empty finite Universe U . The pair (U, R) is called approximation space and R is known as indiscernibility relation.

Let R be a family of equivalence relations on U, then (U, R) be called as knowledge base over U. For $B \subseteq R$, the indiscernibility relation R = IND(B) can be defined as $(x, y) \in IND(B) \Leftrightarrow e(x) = e(y)$ for all e ∈ B and x, y ∈ U. Here e(x) denotes the attribute value of e for the object x.

Definition 3.1. If R be an indiscernibility relation on knowledge U then We define two approximations , for any $X \subseteq U$, $LR(X) = \{y \in U | [y]_R \subseteq X\}$, $HR(X) = \{y \in U | [y]_R \cap X \neq \varphi\}$, are called R- lower approximation and R- Upper approximation of X respectively, where $[y]_R$ is the equivalence class of R contains y.
The set $X \subseteq U$ is called rough set with respect to R if $LR(X) \neq HR(X)$, otherwise the set X is said to be an exact with respect to the knowledge R.

The set POSR(X) = LR(X), NEGR(X) = U − LR(X) and BNDR(X) = HR(X) − LR(X) are called R-positive, R-negative , R- boundary region of X respectively. Also X is said to be a rough set with respect to R , when $BNDR(X) \neq \varphi$.

4. Lattice rough with covering based

Let L be lattice, $R : L \rightarrow L$ be a relation and (L, R) be lattice approximation space.

Definition 4.1. Let $C = \{C_1, C_2, \cdots, C_n\}$ be covering of a L, where C_i is a sublattice of L such that $\cup C_i = L$ for i = 1 to n, and (L,C,R) be the covering lattice approximation space. For $A \subseteq L$, we define two approximation, lower and upper of A as

$LR(A) = \{x \in L | x \in Ci \text{ and } Ci \subseteq A\}$and $HR(A) = L - R(A^c)$. The set A is called lattice rough in L if $LR(A) \neq HR(A)$. Otherwise A is called Lattice definable.

Using above we have,

Theorem 4.2.

For $E, D \subseteq L$, where L is a Lattice,we have

1. $LR(E) \subseteq E \subseteq HR(E)$
2 $LR(\varphi) = HR(\varphi) = \varphi$ and $LR(L) = HR(L) = L$
3. $E \subseteq D \Rightarrow LR(E) \subseteq LR(D)$
4. $E \subseteq D \Rightarrow HR(E) \subseteq HR(D)$
5. $LR(E \cap D) \subseteq LR(E) \cap LR(D)$
6. $LR(E \cup D) \supseteq LR(E) \cup LR(D)$
7. $HR(E \cap D) \subseteq HR(E) \cap HR(D)$
8 $HR(E \cup D) \supseteq HR(E) \cup HR(D)$
9. $LR(-D) = -HR(D)$
10.$HR(-D) = -LR(D)$.

Proof.

1. If $x \in LR(E)$ then $x \in Ci$ for some i and $Ci \subseteq E$, $1 \leq i \leq n$. Hence, $LR(E) \subseteq E$.
Next suppose that $x \in E$ then $x \neq E^c$, so that $x \neq LR(E^c)$ ($\because LR(E^c) \subseteq E^c$)
that is, $x \in L - LR(E^c)$. Thus $x \in HR(E)$, Hence, $E \subseteq HR(E)$.

3. Assume that $E \subseteq D$. If $u \in LR(E)$ then $u \in C_i$ for for some i and $C_i \subseteq E$, $1 \leq i \leq n$.
So that $u \in C_i \subseteq D$. That is, $u \in LR(D)$. Hence, $LR(E) \subseteq LR(D)$

7. Since, $E \cap D \subseteq E$ and $E \cap D \subseteq D$. So, $HR(E \cap D) \subseteq HR(E)$ and $HR(E \cap D) \subseteq HR(D)$
Hence, $HR(E \cap D) \subseteq HR(E) \cap HR(D)$.

Remaining part of proof comes directly.

Example 4.3. Let $L = (P\{a, x, c\}, \subseteq) = \{\varphi, \{a\}, \{x\}, \{c\}, \{a, x\}, \{x, c\}, \{a, c\}, \{a, x, c\}\}$ be a lattice and

$C = \{C_1, C_2, \cdots, C_6\}$ be covering of lattice such that $\cup C_i = L$, for i= 1 to 6.

Let $C_1 = \{\varphi, \{a\}\}$, $C_2 = \{\varphi, \{a\}, \{a, c\}\}$, $C_3 = \{\varphi, \{a\}, \{a, x, c\}\}$, $C_4= \{\{c\}, \{x, c\}\}$, $C_5= \{\varphi, \{c\}, \{a, x\}\}$,

$C_6= \{\{b\}, \{a, x\}\}$ and $A = \{\{a\}, \{c\}, \{x, c\}, \{a, x, c\}\} \subseteq L$. Then

$LR(A) = \{ z \in L | z \in C_i \text{ and } C_i \subseteq A \}.= \{\{c\}, \{x, c\}\}$.

$LR(A^c) = \{z \in L \mid z \in C_i \text{ and } C_i \subseteq A^c\} = \{\{x\}, \{a, x\}\}$

$HR(A) = L - LR(A^c) = \{\varphi, \{a\}, \{c\}, \{a, c\}, \{x, c\}, \{a, x, c\}\}$.

Hence, $LR(A) = HR(A)$. Therefore, A is R− lattice rough.

Definition 4.4. (**Lattice Rough equality of sets**).

For all P, Q $\subseteq$ L, we define a binary relation $\mp$ (bottom lattice R-equal) by P $\mp$ Q if and only if LR(P) = HR(Q), then set P and Q are bottom lattice R− equal.

Also the sets P and Q are top lattice R−equal, we denote P $\pm$ Q if and only if HR(P) = HR(Q). The sets P and Q are lattice R−equal, that is, P $\approx$ Q if and only if P $\mp$ Q and P $\pm$ Q.

Definition 4.5. (**Lattice Rough Inclusion of sets**).

Let L be a lattice. For all P, Q $\subseteq$ L, we define the binary relation $\ll$ (bottom included)
By, P $\ll$ Q $\Leftrightarrow$ LR(P) $\subseteq$ HR(Q), then we say set P is bottom included in Q.

Also set P is top included in set Q that is P $\gg$ Q $\Leftrightarrow$ HR(P) $\subseteq$ HR(Q).

The set P is included in Q that is, P $\cong$ Q $\Leftrightarrow$ P $\ll$ Q and P $\gg$ Q.

Proposition 4.6.

Let P, Q $\subseteq$ L, L be a lattice then we find the following properties

1. If P $\subseteq$ Q then P $\ll$ Q, P $\gg$ Q and P $\cong$ Q.

2. If P $\ll$ Q and Q $\ll$ P then P $\mp$ Q

3. If P $\gg$ Q and Q $\gg$ P then P $\pm$ Q

4. If P $\cong$ Q and Q $\cong$ P then P $\approx$ Q

5. If P $\subseteq$ Q, P $\pm$ $P^/$and Q $\pm$ $Q^/$,then $P^/ \gg$2 $Q^/$

6. If P $\subseteq$ Q, P $\mp$ $P^/$and Q $\mp$ $Q^/$, then $P^/ \ll Q^/$

7. If P $\subseteq$ Q, P $\approx$ $P^/$and Q $\approx$ $Q^/$, then $P^/ \cong Q^/$

8. If $P^/ \gg$ P and $Q^/ \gg$ Q, then $P^/ \cup Q^/ \gg P \cup Q$

9. If $P^/ \ll$ P and $Q^/ \ll$ Q, then $P^/ \cap Q^/ \ll P \cap Q$

10. If P $\gg$ Q and P $\pm$ C , then C $\gg$ Q

11. If P $\cong$ Q and P $\approx$ C, then C $\cong$ Q.

Proof.

1. Given that P $\subseteq$ Q then LR(P) $\subseteq$ LR(Q) $\Leftrightarrow$ P $\ll$ Q.
Also,HR(P) $\subseteq$ HR(Q) $\Leftrightarrow$ P $\gg$ Q.Therefore, P $\cong$ Q.

Remaining proofs come directly.

5. Conclusion

The new mathematical concept of lattice rough set theory is an expansion of the area and scope of application to lattice and rough set theory to manage the difficulties arises on vagueness, impreciseness and dubiousness in our every day life. we first find the covering of a lattice to define covering approximation space on lattice. The new method lattice rough set theory is introduced an some properties are verified.

References

[1] G. Birkhoff, Lattice Theory, Trans. Amer. Math, Soc., New York, 1967.
[2] G. Gratzer, General Lattice Theory, Academic Press, INC, 1978.
[3] D. Mohanty, N. Kalia, L. Pattanayak, B.B. Nayak, An introduction to rough soft set,Mathematical Science, International Research Journal 1(3)(2012) 927-936.
[4] Z. Pawlak, Rough set theory, International Journal of Computer and Information Sciences, 11(1982) 341-356.
[5] Z. Pawlak, Rough sets-theoretical aspects of reasoning about data, Kluwer Academic Publishers 1991.
[6] Y. Shao, Li Fu, F. Hao, K. Qin, Rough Lattice: A Combination with the Lattice theory and Rough set theory, International Conference on Mechatronics, Control and Automation Engineering (2016).